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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED
WITH THE CO-OPERATION OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

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SEPTEMBER, 1910

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THE

PHYSICAL REVIEW.

ON THE DETERMINATION OF THE WAVE-LENGTH AND LOGARITHMIC DECREMENT OF THE LINEAR ELECTRICAL OSCILLATOR WITH THE INTERFEROMETER.

By James E. Ives.

IN a recent paper in the Physical Review,[1] I have described some experiments made with the interferometer to determine the wave-length of the linear electrical oscillator. The receiver was of the thermoelectric type, linear and of low resistance. It was found that the apparent wave-length of the sender depended upon the length of the receiver. In fact, the wave-length curves seemed to be the resultant of two wave-lengths.

As the wave-length of the linear oscillator was still in dispute, being the subject of two rival theories,[2] and as the results I had obtained were not sufficiently definite, I continued my experiments in the endeavor to find a receiver which would give a unique value.

This result I believe I have achieved, after a long and elaborate series of experiments, by using a receiver of very high ohmic resistance.

1. *Apparatus.*—The apparatus used was the same as that described in my last paper,[3] with a few exceptions.

The galvanometer was a two-coil du Bois-Rubens instrument, made by Siemens and Halske,[4] having a resistance, with its two

[1] Vol. 30, pp. 199–215, 1910.
[2] *Loc. cit.*, p. 199.
[3] *Loc. cit.*, pp. 199, 200.
[4] Zeits f. Instrumentenkunde, Jahrg. 1900, p. 65.

coils in parallel of 2.5 ohms, and a sensitiveness of about 1.5×10^{-9} amperes with a period of about 4 seconds. This galvanometer, as used, was about ten times as sensitive as the one used in the previous experiments.

In the receiver the insulated wires, soldered to the fine wires forming the thermoelectric junction, were led back side by side through the center of a wooden rod serving for its support. In the former experiments, one wire was about 3 cm. above the other, producing a difference of temperature between the two wires which disturbed the thermoelectric junction. The iron and constantan wires finally used in the thermal junction were smaller than those formerly used, being only .015 mm. in diameter. They were made by Hartmann and Braun of Frankfurt a. M. At the junction they were welded together by sending a current of about .17 ampere through it. This made the action much more regular.

The three senders were the same as before, and were nearly 5, 7.5 and 10 cm. long respectively. In the course of several weeks' use, they wore away at the spark gap, and their actual lengths, which will be given later, were a little less than these. They were made of brass and had a diameter of .25 cm.

The cylindrical parabolic mirrors used with the 5-cm. sender and receiver had a focal distance of 7.5 cm.; with the 7.5-cm. sender, a focal distance of 11.25 cm.; and with the 10-cm. sender, of 15 cm. These focal distances were so chosen, that if standing waves were formed in front of the mirrors, the sender and receiver would not be in the immediate neighborhood of nodes of electric force. In the previous experiments, one and the same mirror was used for all three senders.

The same separating surface, or grating, was used,[1] but the wires were differently spaced. For the 5-cm. sender, the wires were 1.5 cm. apart, and for the 7.5- and 10-cm. senders, 3 cm. apart. Preliminary experiments seemed to show that for the 5-cm. sender, 1.5 cm. was the spacing which made the reflected and transmitted waves approximately equal, and therefore gave the curve of greatest intensity. A 3-cm. spacing seemed to give better results for the 10-cm. sender than a 6-cm. spacing. Varying the spacings between

[1] *Loc. cit.*, p. 203.

1.5 and 6 cm. did not appear to have any appreciable effect upon the measured values of the wave-lengths of the three oscillators.

2. *Preliminary Experiments.*—To explain the fact that the wave-length curves seemed to be the resultant of two wave-length curves, I suggested, in my last paper,[1] that one component was due to the waves sent out by the sender, and the other to secondary waves sent out by the receiver. If this was so, what was needed was a non-radiative receiver. I therefore constructed such a receiver of copper wire, No. 29, Brown and Sharpe gauge, .0285 cm. in diameter, in the form of a nearly closed circle, open at the top as shown in Fig. 1. The plane of the circuit was placed vertically in the focal line of the parabolic mirror. The free ends of the receiver, a_1 and a_2, were about 2 mm. apart. The thermal junction was at t.

Fig. 1.
Non-radiative receiver.

The experiments were made with the 5-cm. sender, and the total length of the receiver was taken equal to 10, 8, 6, 4.3 cm. respectively. The sets of interferometer readings obtained did not differ materially from those for the straight receivers of the same length shown in Fig. 6 of my last paper.[2] These results therefore show that a so-called non-radiative receiver does not overcome the difficulty. It is interesting to note, in passing, that the ratio of the galvanometer deflections when the receiver was straight to the deflection when it was bent into the form of an open circle, was about 5 to 1. Since a nearly closed circuit is a poor radiator of electrical energy, it will also be a poor absorber, and this ratio will evidently give an approximate measure of the relative radiative power of the two types of oscillator.

As these results seemed to show that the secondary wave-length found in the interferometer curve was not due to the waves sent out by the receiver itself, I made a large number of sets of interferometer readings with the sender and receiver under varying conditions in the endeavor to detect any secondary radiation

[1] *Loc. cit.*, pp. 206–208.
[2] *Loc. cit.*, p. 205.

from the receiver, and to determine the cause of the secondary wave-length. I will only record the most important of these experiments, as a complete record of all would occupy too much space.

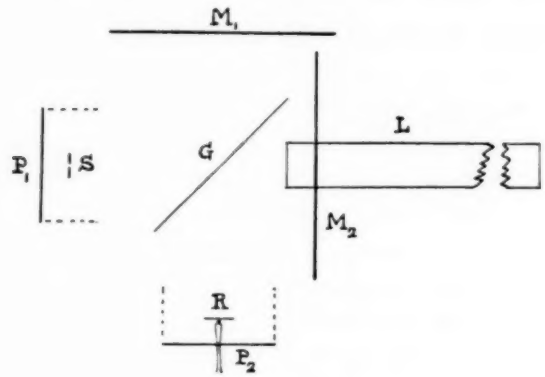

Fig. 2.

Usual arrangement of the apparatus.

The usual arrangement of the apparatus is shown in Fig. 2, where M_1 is the fixed mirror, M_2 movable mirror, S sender, R receiver, L slide, P_1, P_2 parabolic mirrors, G grating.

One experiment consisted in arranging the apparatus as shown in Fig. 3. The sender was 5 cm. long, and the receiver 10 cm., made of no. 29 copper wire. In this case the receiver was excited by the direct wave from S and not, as in Fig. 2, by the reflected waves from M_1 and M_2. It was found that there was a strong excitation of the receiver but no maxima or minima were present in the interferometer curve, as the movable mirror was moved out along the slide. Now, since the receiver was strongly excited, if it had given out waves of its own, comparable in intensity to those given out by the sender, these waves would have produced maxima and minima in the interferometer curve. Since they were not present we must conclude that

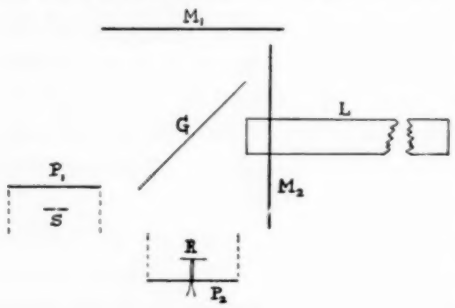

Fig. 3.

Special arrangement of the apparatus.

the secondary waves given out are so weak in comparison with those given out by the sender that they produce no effect upon the interferometer curve within the limits of observation of the present experiment.

Failing to find evidence of radiation from the receiver, it occurred to me that it was possible that the weak oscillations in the receiver or in neighboring conductors might have some sort of a trigger action upon the spark-gap of the sender, and so change its frequency and consequently its wave-length. To test for such an effect I used the 5-cm. sender, and a receiver 4.2 cm. long made of no. 29 copper wire. I first found the interferometer curve in the ordinary manner, and then placed a copper wire of the same diameter and a given length somewhere in the immediate neighborhood either of the sender or of the receiver, and again found the interferometer curve. The lengths and positions of the copper wires were: 10 cm long, placed alongside of the sender in the focal line of the mirror, so that their centers were 12.5 cm. apart; 10 cm. long, 5 cm. in front of sender and parallel to it; 10 cm. long, 6.5 cm. in front of receiver and parallel to it; and 10, 8 and 5 cm, long. respectively, placed 2 cm. in front of receiver, and parallel to it. Within the limits of observation of the experiment, the interferometer curve was found to be the same in each case. These experiments therefore show that, within these limits, the weak oscillations in the receiver and in neighboring conductors, do not exert any direct influence upon the frequency of the sender. They also confirm the conclusion derived from the last experiment described, that the waves sent out by the receiver are not of sufficient intensity to produce any sensible effect upon the interferometer curve. These conclusions follow from the fact that if waves are radiated by the receiver, they must also be radiated by the copper wires, since the total resistance of the copper wires is even less than that of the receiver.

It also seemed to me possible that the resistance of the spark-gap in the sender might, in some way, produce overtones in the sender, which would account for the composite nature of the interferometer curve. Experiments were therefore made with the 5-cm. sender using receivers of different lengths, made of no. 29 copper wire, and varying the length of the spark-gap for the same receiver. Inter-

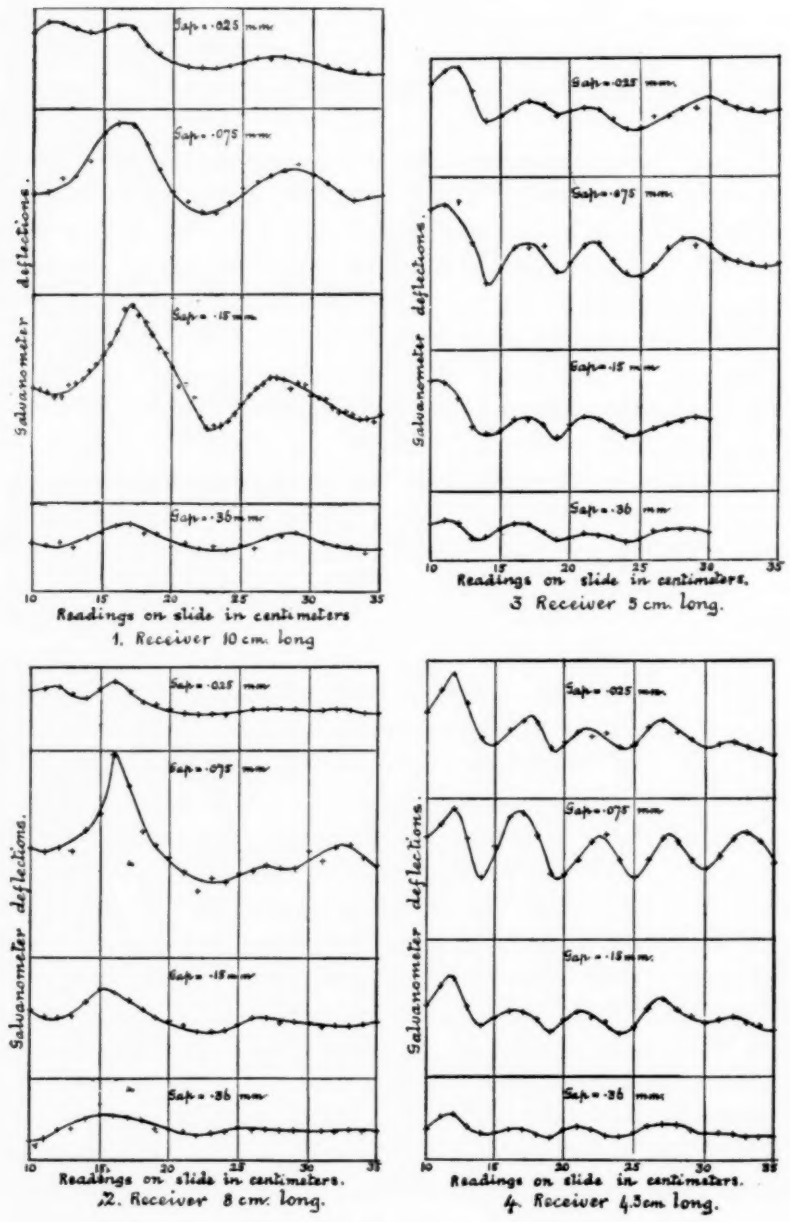
Gap = ·025 mm.
Gap = ·075 mm.
Gap = ·15 mm.
Gap = ·36 mm.
Galvanometer deflections.
10 15 20 25 30 35
Readings on slide in centimeters.
1. Receiver 10 cm. long
3 Receiver 5 cm. long.
2. Receiver 8 cm. long.
4. Receiver 4.3 cm. long.

Fig. 4.

Interferometer curves for the 5-cm. sender, showing the effect of varying the length of the spark-gap with receivers of different lengths.

ferometer curves were obtained for receivers 10, 8, 5 and 4.3 cm. long. In each case curves were obtained for spark-gaps of .36, .15 .075 and .025 mm. respectively. These curves are shown in Fig. 4. It was found that, within the limits of accuracy of the experiment, the form of the interferometer curve depended only upon the length of the receiver, and not upon the length of the spark-gap in the sender. We may therefore assume that, within wide limits, the wave-length of the sender is independent of the length of the spark-gap. Webb and Woodman[1] have come to the same conclusion for the oscillators used by them.

The experiments described above seemed to me to prove that the cause of the dependence of the apparent wave-length of the sender upon the length of the receiver, lay *within* the receiver itself, and not in the field outside of it. If this were so, the cause must be the reaction of the receiver upon the impressed electromotive force. This supposition, as I will show later on, is fully borne out by the theory of the experiment. The only way to get rid of this reaction appeared to me to be to use a dead-beat, or aperiodic receiver. The most evident way to do this was to make the receiver of wire of very high resistance. I therefore constructed the receiver of the same wire that was used for the thermal junction, which, as has already been stated, had a diameter of .00150 cm. I first of all used the iron wire but found later that better results were apparently to be obtained with the constantan wire. I attribute this to the fact, that the ohmic resistance of the constantan wire is about eight times as great as that of the iron wire; and that its permeability is less, making its inductance less. The constantan wire, therefore, as might have been expected proves to be more dead-beat than the iron wire. According to the makers, the ohmic resistances of the constantan and iron wires are 2,850 and 365 ohms per meter, respectively. The receiver was of the same general form as already described. The constantan wires were fastened with a little universal wax to a slip of paper, which was attached to a small wooden board with a little sealing wax. The end of one of the constantan wires was soldered to an iron wire of the same diameter. This iron wire and the end of the other constantan wire were crossed, forming the ther-

[1]PHYS. REV., 29, p. 130, 1909.

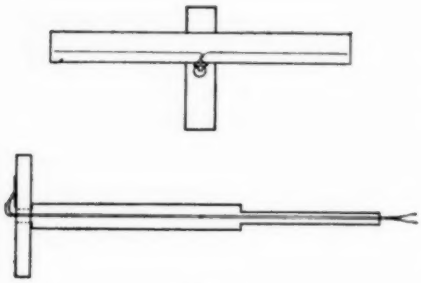

Fig. 5.
High resistance receiver.

mal junction, and their ends soldered to the wires leading to the galvanometer. The front and side views of this receiver are shown in Fig. 5.

3. *Measurements of the Wave-lengths of the Oscillators with HighResistance Receivers.* —Fig. 6 shows the interferometer curves obtained for the 5-cm. sender using high resistance receivers of different lengths made of constantan wire, .00150 cm. in diameter.

It will be noticed that the apparent wave-length is nearly independent of the length of the receiver. There is a remarkable con-

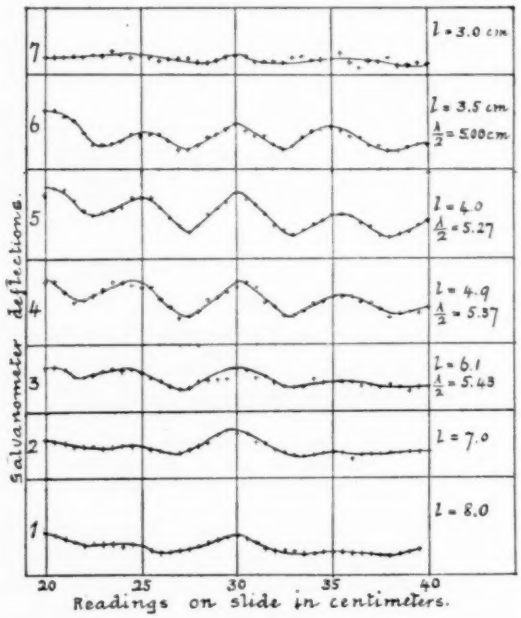

Fig. 6.

Interferometer curves for the 5-cm. sender obtained with high resistance receivers of constantan wire of different lengths.

trast in this respect between these curves, and those shown in Fig. 6 of my last paper[1] where the receiver used was of low resistance.

[1] *Loc. cit.*, p. 205.

In the present experiments, the mirrors were equidistant from the grating when the reading on the slide was 30 cm., so that the maximum occurring at this reading is the central maximum. The fact that the readings to the left of the central maximum were usually greater than those to the right was, in this case, in the main due to the deterioration of the sender at the spark-gap as the time went on. The readings were usually taken from left to right, moving the mirror away from the grating. The spark-gap was adjusted at the beginning, and not changed during a set of readings. After the set was completed certain check readings were taken moving the mirror backwards, from right to left. These readings are indicated by circles, and give a measure of the effect of the deterioration of the spark-gap. The actual length of the 5-cm. sender including the spark-gap was 4.93 cm. The curves show that the receiver is not entirely dead-beat, as the excitation is greatest for a receiver of a certain length. The length of the receiver was varied from 8 to 3 cm. In connection with each curve is given the length, l, of the receiver used, and the half wave-length $\lambda/2$, obtained from the central maximum and the first maximum and minimum to the right and left. Curve 5 appears to be the one in which the maxima and minima are most strongly developed and evenly spaced. From this curve, we get $\lambda/2 = 5.27$ cm. Taking the mean of curves 4, 5 and 6, we get $\lambda/2 = 5.21$ cm. Since the actual length of the 5-cm. sender, including the spark-gap, was 4.93 cm., the correction for its width calculated from Abraham's formula[1] is .13. The half wave-length, therefore, according to this formula should be 5.06 cm. The experimental and calculated values therefore differ by about 3 per cent. The ratio of the wave-length, found by experiment, to the length of the oscillator, is 2.10, or the wave-length was about 5 per cent. greater than twice the length of the oscillator. The curves obtained with the receiver made of *iron* wire will be discussed later.

Fig. 7 shows the interferometer curves for the 7.5-cm. oscillator, using the constantan wire receiver, .00150 cm. in diameter. The length of the receiver was varied from 10 to 5 cm. Curve 4 may be selected as the one in which the maxima and minima are most

[1] PHYS. REV., 30, p. 202, 1910.

clearly defined and equally spaced. It probably represents the length of receiver most nearly in resonance with the sender. It

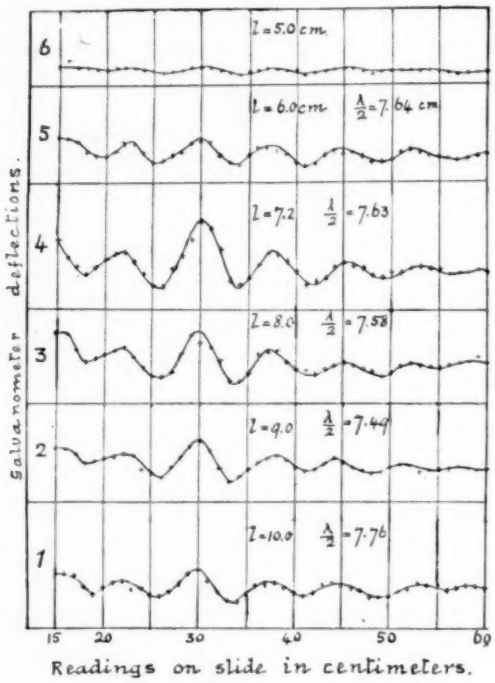

Fig. 7.

Interferometer curves for the 7.5-cm. sender obtained with high resistance receivers of constantan wire of different lengths.

gives $\lambda/2 = 7.63$ cm. Taking the mean of $\lambda/2$ for all six curves we get 7.62 cm.

The length of the 7.5-cm. sender including the spark-gap was actually 7.49 cm. The correction to be added to this, according to Abraham's formula, is .16 cm. The half wave-length, then, according to this formula, should be 7.65 cm. The experimental and calculated values therefore agree within the experimental errors. The ratio of the wave-length to the length of the oscillator is equal to 2.036, or the wave-length is about 2 per cent. greater than twice the length of the oscillator.

Fig. 8 shows the curves obtained for the 10-cm. sender. Curve no. 5 is apparently the one in which the maxima and minima are most clearly marked, and evenly distributed. This curve gives $\lambda/2 = 10$ cm. The mean $\lambda/2$ given by all the curves is 9.93 cm.

The actual length of the 10-cm. sender, including the spark-gap was 9.85 cm. The correction to be added, according to Abraham's

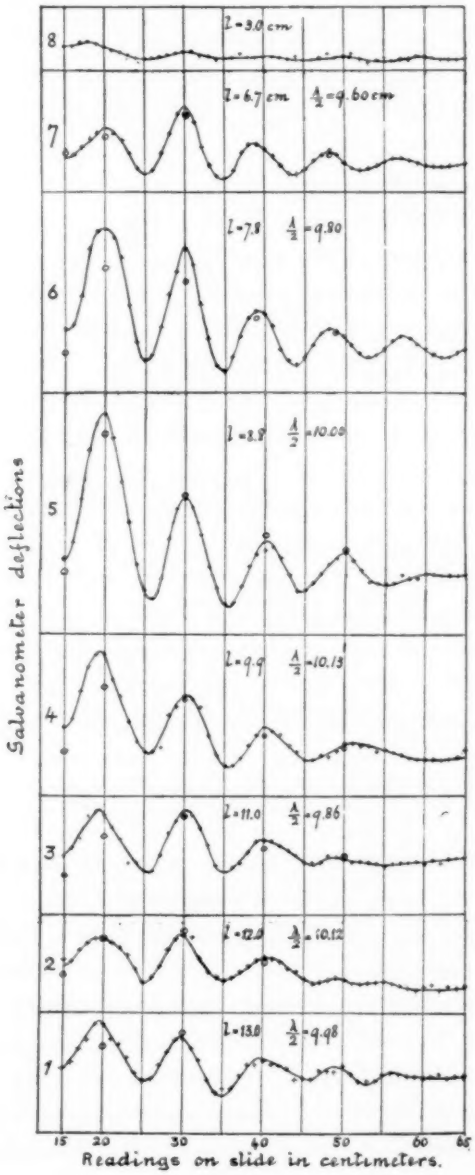

Fig. 8.

Interferometer curves for the 10-cm. sender obtained with high resistance receivers of constantan wire of different lengths.

formula, is .18 cm., and the half wave-length calculated from this formula is 10.03 cm. Taking the most probable value of the half wave-length as that given by curve 5, we get a very close agreement between the experimental and calculated values. Even if we take the mean value obtained from all the curves, the agreement is still within one per cent. The ratio of the wave-length to the length of the oscillator is 2.030, or the wave-length is about 1.5 per cent. greater than twice the length of the oscillator.

In Figs. 6–8 the maximum occurring at about 30 cm. is the central maximum, and ought to be higher than the maxima on both sides of it. In some cases this is not so for the maximum to the left of it. This is due sometimes to the decrease in the strength of the radiation as the time goes on, but not always. This is shown by the check readings taken after the set of readings was completed, and given in the figure by circles. These check readings show that the first maximum to the left of the central maximum is often higher than the central maximum itself. I have not been able to explain this fact. It appears to be due in some way to the presence of the parabolic mirrors, as it disappears when the mirrors are taken away, as shown in Fig. 9, curves 1 and 2. I also recorded the same effect in my last paper[1] in Figs. 12 and 13.

In Figs. 6–8 the curves were all plotted to the same scale with the exception of 6 and 7, Fig. 8, in which the unit used for the ordinates was only half as large. If they were plotted to the same scale as the others their ordinates would be twice as high as they are.

In my previous paper[2] I discussed and made some experiments upon the effect of the oil, and hard-rubber cylinder, surrounding the spark-gap, upon the wave-length of the sender. Further experiments were made upon this point. The difficulty encountered when investigating this matter, is that when the oil is taken away from the spark-gap, the oscillations in the sender, and consequently its radiation, become very weak. I therefore used the 10-cm. sender in these experiments, as it radiates more energy than the shorter oscillators. The results are shown in Fig. 9. The wave-length was first measured with the sender supported in the hard-rubber cup

[1] *Loc. cit.*, pp. 212, 213.

[2] *Loc. cit.*, p. 208.

without any oil. The results are given in curves 1 and 2. They show that within the limits of accuracy of the experiment the presence of the oil has no effect upon the wave-length. There still remains the possible effect of the hard-rubber cylinder supporting the sender. The brass cylinders forming the oscillator were therefore

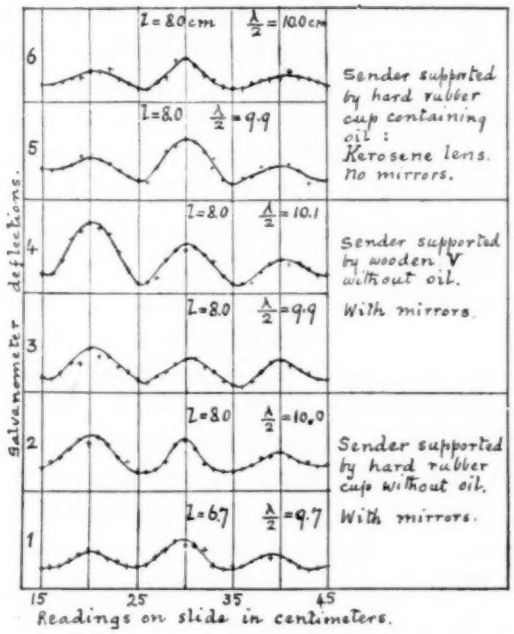

Fig. 9.

Interferometer curves, for the 10-cm. sender, made to determine the effect of the kerosene oil, the hard-rubber cap and the mirrors upon its wave-length.

next supported by a wooden V of the form shown in Fig. 10, and the wave-length determined. 3 and 4, Fig. 9, are the curves obtained. In this case, both the oil and the hard-rubber cylinder were removed from the neighborhood of the spark-gap. It will be seen from the curves that, within the limits of accuracy of the experiment, the wave-length of the oscillator is the same when one or both of them are absent as when they are present. In other words, for the 10-cm. oscillator used in these experiments, the wave-length obtained when sparking in oil in the hard-rubber cylinder, is the same as that when sparking free in air.

The experiments just described were made with the sender and receiver placed in the foci of parabolic mirrors. It does not seem

probable that the presence of the mirror, which is a thin insulated sheet of zinc, could change appreciably the capacity of the oscillator, and consequently change its wave-length, but it is conceivable that a concave mirror may have a period of its own just as a hollow metal cylinder has a period of its own within its cavity,[1] which might effect in some way the wave-length of a linear oscillator placed at its focus. To determine, then, whether the mirrors had any effect upon the wave-length of the 10-cm. sender, both mirrors were removed, and the kerosene lens, described in my last paper,[2] placed in front of the sender to concentrate the energy upon the receiver. The sender in this case was supported in the hard-rubber cup containing the kerosene oil around the spark-gap. Curves 5 and 6 of Fig. 9 were obtained. It will be seen that the wave-length given by these curves is, within the limits of accuracy of the experiment, the same as that obtained when the mirrors were used. We may therefore assume that the presence of the mirrors does not have any appreciabe effect upon the wave-length of the 10-cm. linear oscillator. Since the lengths of the 5- and 7.5-cm. oscillators are of the same order as that of the 10-cm. oscillator, we may, I think, conclude that the wave-lengths determined for all of them are very nearly equal to their true wave-lengths when oscillating free in air, and that these wave-lengths are not affected to any great degree by the presence of the oil around the spark-gap, the hard-rubber cylinder or the mirrors. The results obtained seem to show that the error due to these causes must be less than 1 per cent. for the 10-cm. oscillator.

Fig. 10.

10-cm. sender supported by a wooden V.

4. *The Theory of the Interferometer Method for Damped Oscillations.*—The theory of the interferometer as used in experiments on light assumes that the receiver is dead-beat, and that the wave-train emitted by the source is undamped. The case of the measurement of electrical wave-lengths is quite different, for, in general, the receiver is not dead-beat, and the sender is strongly damped.

Bjerknes[3] was the first to draw attention to the important effect

[1] Winkelmann's Handbuch, 2d ed., Bd. V., p. 676.

[2] *Loc. cit.*, p. 211.

[3] See Ann. Phys., 44, 1891, pp. 74–91, 92–101, 513–526; 47, 1892, pp. 69–76; 54, 1895, pp. 58–63; 55, 1895, pp. 121–169.

of the damping of the sender and receiver upon the measurement of wave-lengths, and upon resonance curves, and to show how it explained the so-called Multiple Resonance of Sarasin and De La Rive.

Bartenstein[1] in his recent discussion of the effect of the damping of an oscillator upon the diffraction of electric waves, has given an excellent discussion of the form of such a wave-train.

In the interferometer, as used in our experiment, we have two damped wave-trains differing in phase falling upon a receiver which has a period of its own. Two cases are easy to visualize. In the first, the receiver is dead-beat and the sender oscillates. In this case the distance between the maxima of the interferometer curve will be equal to half of the *wave-length of the oscillator*. In the second case, the sender is dead-beat, and the receiver oscillates. In this case it is evident that the distance between two maxima of the interferometer curve will be equal to half of the *wave-length of the receiver*. The difficulty is to form a mental picture of what happens when both the sender and receiver oscillate with their own periods and dampings.

Bjerknes[2] has developed the theory for the standing waves produced on two long parallel wires by damped trains of waves sent along them and reflected from their ends, when the length of the wires is great compared with the length of the waves. Hull[3] has indicated how this theory may be modified so as to apply to the interferometer.

The difference between the case treated by Bjerknes, and that of the interferometer, is that in the first we have interference between direct and reflected waves moving in opposite directions, and in the second, between two reflected waves moving in the same direction. In one case we have standing waves with respect to space, and in the other, with respect to time. In the first, at any given instant, the electric force varies from point to point in the neighborhood of the receiver, whereas, in the second, the electric field is the same at *every point* in its neighborhood.

The theory of the interferometer method is the same as that of

[1]Ann. Phys., 29, pp. 225–227, 1909.
[2]Ann. Phys., 44, pp. 513–526, 1891.
[3]Phys. Rev., 5, pp. 231–246, 1897.

the two-mirror method of Boltzmann used by Klemencic and Czermak,[1] and more recently by Webb and Woodman.[2]

In the development of the theory, which will now be given, the general method of Bjerknes will be followed.

In our interferometer, we have interference between two damped trains of waves differing in phase. We will therefore first attempt to find the differential equation for the disturbance in the receiver produced by a single train, and then find the integral effect of the two trains acting together. Since the thermal junction is placed at the middle point of the receiver, and the galvanometer deflection is due to the thermoelectric current produced by the heating of this junction, and the heating effect is proportional to the square of the current, it is necessary to determine the value of the current at this point.

The receiver used in these experiments had a very high ohmic resistance, which must be taken into account in the theory of its action. As far as I am aware no complete theory of the *high resistance linear oscillator* has been worked out.

An approximate theory may, be developed by considering the high resistance linear oscillator as a system with one degree of freedom, since it is well known that when electric waves disturb such an oscillator they produce on it standing waves. For its fundamental mode of vibration we have a current loop at its middle point, and potential loops at its ends. The density of the charge on the oscillator and the current in it therefore vary from point to point along it, and its inductance, resistance and capacity cannot be measured in the ordinary way. Measuring from the middle point of the oscillator, the potential is proportional to a sine function of the distance x, and the current to a cosine function of this distance. Fleming has shown[3] that the capacity of such a linear oscillator is equal to $2/\pi$ times its usual value, calculated on the assumption that the charge is uniformly distributed along the whole length of the wire. We may therefore write

$$C' = 2/\pi \cdot C, \tag{1a}$$

[1] Ann. Phys., 50, pp. 174–188, 1893.
[2] PHYS. REV., pp. 29, 89–131, 1909.
[3] Electric Wave Telegraphy, p. 194.

where C is the capacity calculated in the usual way, and C' is the effective capacity.

In the same way we may show that the effective inductance and resistance are given by

$$L' = 2/\pi \cdot L, \tag{1b}$$

$$R' = 2/\pi \cdot R. \tag{1c}$$

We must also remember that in a linear oscillator the total dissipation of energy is due not only to its ohmic resistance, but also to its radiation.

Fleming[1] has shown that the rate at which energy is radiated from a linear oscillator is given, in watts, by

$$\frac{dW}{dt} = 80\pi^2 \frac{l^2}{\lambda^2} i^2, \tag{1d}$$

where l is the length of the oscillator, λ wave-length, i current at its center.

Therefore the rate at which energy is converted into heat, and the rate at which it is radiated from the oscillator are both proportional to the square of the current. We may therefore write, calling the dissipation constant R'',

$$R'' = R' + \zeta, \tag{1e}$$

where ζ is that part of the dissipation constant due to radiation, and is given by

$$\zeta = \frac{80\pi^2 l^2 \times 10^7}{\lambda^2}. \tag{1f}$$

The total rate of dissipation of energy is therefore given by

$$dW/dt = (R' + \zeta)i^2 = R''i^2. \tag{1g}$$

It is evident that, if the ohmic resistance is small compared with the radiation,

$$dW/dt = \zeta i^2.$$

If great compared with the radiation,

$$dW/dt = R'i^2.$$

Forming the equation of the electromotive forces for the oscillator,

[1]Radiotelegraphy, p. 140.

we have

$$\frac{d(L'i)}{dt} + R''\imath + \frac{q}{C'} = Ee^{-at} \sin\left(at - \frac{\pi}{2}\right) = -Ee^{-at} \cos at,$$

where $Ee^{-at} \sin (at - \pi/2)$ is the electromotive force impressed upon the receiver by the impinging electric waves, assumed to lag 90° behind the electric field; i is the current at its center, and q the quantity of electricity on one half of it. α is the damping factor of the sender, and a is equal to 2π times its frequency.

This equation may be written in the form

$$(2) \quad \frac{d^2i}{dt^2} + \frac{R''}{L'}\frac{di}{dt} + \frac{i}{L'C'} = \frac{E(\alpha^2 + a^2)^{\frac{1}{2}}}{L'} e^{-at} \sin\left(at + \tan^{-1}\frac{\alpha}{a}\right).$$

Since $\alpha = \delta n$ where δ is the logarithmic decrement of the sender, and n is its frequency, and $a = 2\pi n$,

$$\alpha/a = \delta/2\pi.$$

For each of the three oscillators used, as will be shown in the next section, δ was of the order of 1. Therefore $\tan^{-1} \alpha/a$ is of the order of 9°, and the lag is not large. We will therefore neglect $\tan^{-1} \alpha/a$.

For convenience we will write

$$\frac{R''}{L'} = 2\beta, \quad \frac{1}{L'C'} = b^2 + \beta^2 \quad \text{and} \quad \frac{E(\alpha^2 + a^2)^{\frac{1}{2}}}{L'} = A;$$

(2) then becomes

$$(3) \qquad \frac{d^2i}{dt^2} + 2\beta\frac{di}{dt} + (b^2 + \beta^2)i = Ae^{-at} \sin at.$$

The initial conditions are

$$(4) \qquad t = 0, \quad i = 0, \quad di/dt = 0.$$

The complete solution of (3) is given by

$$(5) \qquad i = A_1e^{-at} \sin (at + \epsilon_1) + B_1e^{-\beta t} \sin (bt + \epsilon_2).$$

where

$$A_1 = \frac{[[(\alpha-\beta)^2-(a^2-b^2)]^2+4a^2(\alpha-\beta)^2]^{\frac{1}{2}}}{[(\alpha-\beta)^2-(a^2-b^2)]^2+4a^2(\alpha-\beta)^2}A,$$

$$B_1 = \frac{[a^2/b^2[(\alpha-\beta)^2+(a^2-b^2)]^2+4a^2(\alpha-\beta)^2]^{\frac{1}{2}}}{[(\alpha-\beta)^2-(a^2-b^2)]^2+4a^2(\alpha-\beta)^2}A,$$

$$\epsilon_1 = \tan^{-1}\frac{2a(\alpha-\beta)}{(\alpha-\beta)^2-(a^2-b^2)},$$

$$\epsilon_2 = \tan^{-1}\frac{-2b(\alpha-\beta)}{(\alpha-\beta)^2+(a^2-b^2)}.$$

The current in the receiver is therefore the sum of two damped sine functions of the time, one having the period and damping of the sender, and the other those of the free vibrations of the receiver.[1] The relative amplitudes of the two vibrations depend upon the relation of their damping factors and periods.

Since

$$b = \sqrt{\frac{1}{L'C'}-\frac{R''^2}{4L'^2}}$$

the receiver will cease to oscillate *with its own period* when

$$R''^2 = 4L'/C'. \tag{5a}$$

When R'' is greater than this value the period of the oscillation in the receiver will be equal to that of the sender.

The frequency of the free vibration of the receiver is equal to $b/2\pi$. Therefore its period, T, is given by

$$T = \frac{2\pi}{\sqrt{\frac{1}{L'C'}-\frac{R''^2}{4L'^2}}}. \tag{5b}$$

When R'' is very small, this reduces to the well-known approximate formula for the linear oscillator

$$T = 2\sqrt{LC}. \tag{5c}$$

The heat developed at the thermal junction is proportional to

[1]See Larmor, Æther and Matter, pp. 241–243.

the integral

$$\int_{t=0}^{t=\infty} R_m i^2 dt,$$

where R_m is the resistance of the wire forming the thermal junction. The total effect is due to the two trains of waves, coming from the fixed and movable mirrors, respectively. If x is the distance, on the slide, of the movable mirror from its position when both mirrors are equidistant from the grating, the train from the movable mirror will arrive ahead of that from the fixed, by a time equal to $\pm 2x/v$, where v is the velocity of propagation of electric waves in space. If I is the integral heating effect at the thermal junction measured by the deflection of the galvanometer, and we write $i = \phi(t)$, we have for any given position x of the movable mirror

$$(6) \qquad I = c\int_{t=0}^{t=2x/v} \phi^2(t)dt + c\int_{t=0}^{t=\infty} \left[\phi\left(t + \frac{2x}{v}\right) + \phi(t)\right]^2 dt,$$

where c is a constant depending upon the nature, dimensions and arrangement of the apparatus used, and $\phi(t) = i$ is given by (5). Putting $c = 1$, (6) may be written

$$(7) \qquad I = 2K + 2\int_{t=0}^{t=\infty} \phi\left(t + \frac{2x}{v}\right)\phi(t)dt,$$

where

$$K = \int_{t=0}^{t=\infty} \phi^2(t)dt.$$

The problem is to determine the value of the second term of the right-hand member of (7) in terms of the distance x.

Substituting the values of $\phi(t + 2x/v)$ and $\phi(t)$, and writing $2x/v = \tau$, we get

$$\int_{t=0}^{t=\infty} \phi(t+\tau)\,\phi(t)dt$$

$$= \int_{t=0}^{t=\infty} \{A_1 e^{-\alpha(t+\tau)} \sin[a(t+\tau)+\epsilon_1] + B_1 e^{-\beta(t+\tau)} \sin[b(t+\tau)+\epsilon_2]\}$$

$$[A_1 e^{-\alpha t} \sin[at+\epsilon_1] + B_1 e^{-\beta t} \sin[bt+\epsilon_2]]dt$$

$$= A_1^2 \int_{t=0}^{t=\infty} e^{-\alpha\tau} e^{-\alpha(t+\tau)} \sin(at+\epsilon_1) \sin(at+\epsilon_1+a\tau)dt$$

$$
\begin{aligned}
&+ A_1B_1 \int_{t=0}^{t=\infty} e^{-at}e^{-\beta(t+\tau)} \sin(at+\epsilon_1) \sin(bt+\epsilon_2+b\tau)dt \\
&+ A_1B_1 \int_{t=0}^{t=\infty} e^{-\beta t}e^{-a(t+\tau)} \sin(bt+\epsilon_2) \sin(at+\epsilon_1+a\tau)dt \\
&+ B_1^2 \int_{t=0}^{t=\infty} e^{-\beta t}e^{-\beta(t+\tau)} \sin(bt+\epsilon_2) \sin(bt+\epsilon_2+b\tau)dt.
\end{aligned} \tag{8}
$$

The third integral of (8) is equal to

$$
\begin{aligned}
&\frac{e^{-a\tau}}{2}\Bigg[\frac{\alpha+\beta}{(\alpha+\beta)^2+(a-b)^2}\{\cos a\tau\cos(\epsilon_1-\epsilon_2)-\sin a\tau(\epsilon_1-\epsilon_2)\} \\
&\quad-\frac{a-b}{(\alpha+\beta)^2+(a-b)^2}\{\sin a\tau\cos(\epsilon_1-\epsilon_2)+\cos a\tau\sin(\epsilon_1-\epsilon_2)\} \\
&\quad-\frac{\alpha+\beta}{(\alpha+\beta)^2+(a+b)^2}\{\cos a\tau\cos(\epsilon_1-\epsilon_2)-\sin a\tau\sin(\epsilon_1+\epsilon_2)\} \\
&\quad+\frac{a+b}{(\alpha+\beta)^2+(a+b)^2}\{\sin a\tau\cos(\epsilon_1-\epsilon_2)+\cos a\tau\sin(\epsilon_1+\epsilon_2)\}\Bigg].
\end{aligned} \tag{9}
$$

The other three integrals of (8) are of the same form, and (8) may therefore be written

$$
= \frac{e^{-a\tau}}{2} D_1 \cos(a\tau-\epsilon_3) + \frac{e^{-\beta\tau}}{2} D_2 \cos(b\tau-\epsilon_4), \tag{10}
$$

where D_1, D_2, ϵ_3 and ϵ_4 are functions of a, b, α, β and A, are independent of τ, and may be determined from the four integrals of (8).

Substituting the value of τ we finally get for I

$$
I = 2K + e^{-2ax/v} D_1 \cos\left(\frac{2ax}{v}-\epsilon_3\right) + e^{-2\beta x/v} D_2 \cos\left(\frac{2bx}{v}-\epsilon_4\right). \tag{11}
$$

It is seen that, in general, the interferometer curve is the resultant of two damped cosine functions of x, one having the natural period and damping of the sender, and the other of the receiver. This is what the experimental results, published in my last paper,[1] led me to suppose, and the mathematical analysis just given shows that this supposition was correct.

(11) shows that the logarithmic decrement of each component of the interferometer curve is the same as that of the sender and

[1] *Loc. cit.*, p. 207.

receiver respectively. Also that the wave-length of each component is equal to half of the wave-length of the sender and receiver respectively.

If the resistance of the receiver is great compared with that of the sender, we have β large compared with α, a and b, and the receiver component becomes small compared with the sender component. In this case (11) becomes

$$I = 2K + e^{-2ax/v} D_1 \cos\left(\frac{2ax}{v} - \epsilon_3\right). \tag{12}$$

This is approximately the case of the present experiment, and we will consider it by itself. Under these conditions, (5) becomes

$$i = \frac{A}{\beta^2} e^{-at} \sin at \tag{13}$$

and

$$I = \frac{1}{2} \frac{a^2 A^2}{\alpha(\alpha^2+a^2)\beta^4} \left[1 + e^{-2ax/v} \frac{[\alpha^2+a^2]^{\frac{1}{2}}}{a} \cos\left(\frac{2ax}{v} - \tan^{-1}\frac{\alpha}{a}\right)\right]. \tag{14}$$

(14) gives the form of the interferometer curve when the receiver has a very high resistance and is approximately dead-beat. It is seen that, in general, there is a lag, equal to $\tan^{-1} \alpha/a$. This lag will displace the first maximum to the right and will tend to make $\lambda/2$ as measured between the central maximum and the first maximum too great.

If the damping of the sender is small, the lag may be neglected, and (14) may be written

$$I = \frac{A^2}{2\alpha\beta^4}\left[1 + e^{-2ax/v} \cos\frac{2ax}{v}\right]. \tag{15}$$

The ordinate of the interferometer curve is then made up of two parts, one a constant part $= 2K = A^2/2\alpha\beta^4$, and the other a damped cosine function of the distance x measured along the slide. The ordinate of the central maximum is equal to twice the constant part, that is, to $A^2/\alpha\beta^4$. After the oscillation of the curve has died away the ordinate assumes the constant value $A^2/2\alpha\beta^4$. The logarithmic decrement of this curve is equal to the logarithmic decrement of the sender, and its wave-length to half of the wave-length of the sender.

It has been pointed out by Bjerknes,[1] for the case of standing waves on long wires, that the interference curve obtained with a dead-beat receiver gives a *complete picture* of the wave-train emitted by the sender. It is evident that this is also true of the interferometer curve obtained with a dead-beat receiver.

The curves of Figs. 6–8 confirm (15) fairly well.

In Fig. 11 are plotted more extensive series of readings than are given in the other curves. They show, among other things, the

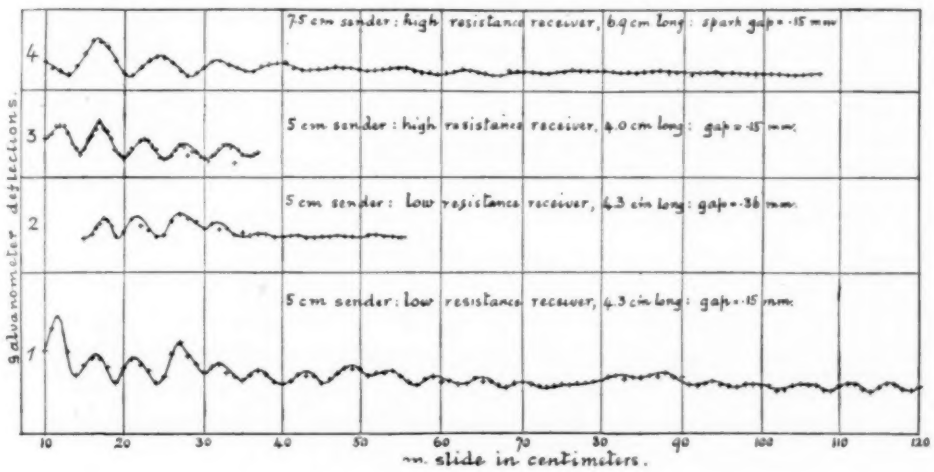

Fig. 11.

Interferometer curves giving extensive series of readings.

difference in the curves obtained when using low- and high-resistance receivers, and the effect of the damping due to the spark-gap.

1 and 2 were obtained with a low-resistance receiver made of no. 29 copper wire.[2] 3 and 4 with high-resistance receivers; 3 with a constantan wire .0015 cm. in diameter, and 4 an iron wire of the same diameter. 1, 2 and 3 were obtained with the 5-cm. sender, and 4 with the 7.5-cm. sender. For a similar curve for the 10-cm. sender, the reader is referred to 5 of Fig. 8. 1, 3 and 4 were obtained with a .15-mm. spark-gap, and 2 with a .36-mm. gap. The central maximum of each curve was in the neighborhood of the 17-cm. reading on the slide.

It will be interesting to calculate from the theory just given the

[1]Ann. Phys., 44, 515, 1891.

[2]PHYS. REV., 30, 200, 1910.

ohmic resistance necessary to make the receiver aperiodic. In this case the radiation vanishes, since the wave-length becomes infinite and $R'' = R'$. From (5a) we then have

$$R' = 2\sqrt{L'C'},$$

where

$$R' = 2/\pi \cdot R,$$
$$L' = 2/\pi \cdot L = 2/\pi \times 2l[\log_e 2l/r - 1],$$
$$C' = \frac{2}{\pi}C = \frac{2}{\pi} \times \frac{1}{8} \frac{l}{v^2 \log_e \frac{l}{2r}}.$$

(It will be noticed that we take C equal to one half of the distributed capacity of one half of the oscillator.[1])

We then have for the value of the resistance R which will make the linear oscillator dead-beat,

$$R = \pi\sqrt{L/C}.$$

For the 10- and 5-cm. receivers made of constantan wire R would be equal to 3,400 and 3,100 ohms, respectively. The actual resistances of the receivers used were about 285 and 143 ohms, respectively, so that they were considerably below the values necessary to make them completely aperiodic according to the theory given above.

5. *The Logarithmic Decrement.*—The logarithmic decrement of the sender can be found from the interferometer curve if we use a dead-beat receiver. Assuming that the high-resistance receiver used in these experiments was nearly dead-beat, we can calculate the decrements of the 5-, 7.5- and 10-cm. senders from curve 5, Fig. 6; 4, Fig. 7, and 5, Fig. 8, using the formula

$$\delta = 2 \log_e \left[\frac{a_1 - a_2}{a_3 - a_2}\right],$$

where a_1, a_2 and a_3 are the ordinates of the central maximum, and first minimum and maximum to the right. Doing so, we get

5-cm. sender, δ = 1.26,
7.5-cm. sender, 1.15,
10-cm. sender, .95.

[1]See Hertz, Electric Waves, supplementary note, No. 6, p. 270.

The decrement decreases as the length of the oscillator increases, as would be expected. It is evident, however, from the nature of the experiment and the sources of error entering into it, that these values are only approximate. The determination of the decrement, however, from the other curves, using one or several oscillations, shows that they are of this order.

These decrements include not only the decrement due to radiation, but also that due to the resistance of the spark-gap. The part due to the ohmic resistance of the oscillator itself can evidently be neglected.

Some idea of the effect of the length of the spark-gap upon the magnitude of the total decrement can be obtained from the two

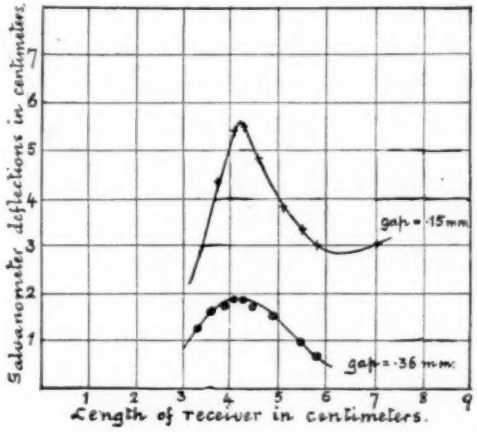

Fig. 12.

Resonance curves for the 5-cm. sender, obtained with a low resistance receiver, using spark-gaps of different lengths.

tuning curves for the 5-cm. sender shown in Fig. 12. They were obtained by taking the length of the spark-gap equal successively to .036 cm. and .015 cm. The receiver used in this case was the low-resistance receiver, already described, made of no. 29 copper wire.

If δ_1 is the decrement of the sender, δ_2 that of the receiver, then we have by Bjerknes's[1] formula

$$\delta_1 + \delta_2 \frac{2\pi}{X}\sqrt{\frac{abc}{d}},$$

[1]Ann. Phys., 55, p. 149, 1895.

where a and b are the segments of a horizontal cord, and d and c are the upper and lower segments of the ordinate drawn to the isochronous point; and X is the abscissa of this ordinate. From this formula we get for Fig. 12

$$\text{Spark-gap, .015 cm.} \quad \delta_1 + \delta_2 = 1.50,$$
$$\text{Spark-gap, .036 cm.} \quad \delta_1 + \delta_2 = 1.96.$$

From the interferometer curve, using the high resistance receiver, we have already found $\delta_1 = 1.26$. Therefore $\delta_2 = .24$. Subtracting this value from $\delta_1 + \delta_2$ in each case we get

$$\text{Spark-gap, .015 cm.} \quad \delta_1 = 1.26,$$
$$\text{Spark-gap, .036 cm.} \quad \delta_1 = 1.72.$$

If we assume that for small spark-gaps the decrement due to the spark is a linear function of the length of the gap, we may write

$$\delta_1 = \delta_r + \delta_c z, \tag{17}$$

where δ_r is the radiation decrement, δ_c is the decrement for a gap 1 cm. long and z is the length of the gap.

Solving (17) for δ_r and δ_c we get

$$\delta_r = .93,$$
$$\delta_c = 22.$$

For the 5-cm. sender we then obtain a radiation decrement = .93. For the spark-gap decrement, δ_s, we obtain

$$\text{Spark-gap, .015 cm.} \quad \delta_s = .33,$$
$$\text{Spark-gap, .036 cm.} \quad \delta_s = .79.$$

Abraham[1] has given the following formula, as a second approximation, for the radiation decrement of a linear oscillator, having no ohmic resistance.

$$\delta_r = 9.74\epsilon + 47.6\epsilon^2,$$

where

$$\epsilon = \frac{1}{4 \log_e \frac{l}{r}},$$

where l is the length of the oscillator and r its radius.

[1]Ann. Phys., 66, p. 464, 1898.

Calculating the decrement, from this formula, for the three senders, used in these experiments, we get

$$l = 4.91 \text{ cm.} \quad \delta_r = .882,$$
$$l = 7.47 \text{ cm.} \quad \delta_r = .771,$$
$$l = 9.83 \text{ cm.} \quad \delta_r = .712.$$

The value of .88 for the radiation decrement of the 5-cm. sender, calculated from Abraham's formula, therefore agrees, fairly well with the value of .93 found by experiment.

6. *Miscellaneous Matters.*—It is evident that there are two trains of waves reaching the grating, one consisting of cylindrical waves coming direct from the sender, and the other, of plane waves, by reflection from the parabolic mirror. It is also evident that the plane waves must be most effective in forming the interferometer curve, but the question arose as to how far this curve was affected by the reflected waves. To investigate this point, shields of sheet zinc, having the form of segments of cylinders, were placed in front of the sender to as to reflect back to the mirror, through its focal line, practically all of the direct wave. For the 5-cm. sender, the shield was 15 cm. long, 6 cm. wide and had a radius of 5 cm.; for the 7.5-cm. sender, 27 cm. long, 10.4 cm. wide and 5 cm. radius. Various measurements of the wave-length were obtained with and without the shields. It was found that the presence of the shield did not affect the value of the wave-length as determined from the interferometer curve, and also that its presence or absence did not have any marked effect, one way or the other, upon the intensity of the received energy. The direct wave, therefore, does not appear to produce any sensible disturbing effect upon the shape of the interferometer curve, and the use of these shields does not seem to be necessary.

The effect of reflection from the wall of the room was determined by taking away the fixed mirror. The wall then acted as the fixed mirror, and a very marked reflection was found. However, since the wave-length measured was short, the distance traversed by the waves not great, and the plane mirrors, large, I think that the effect upon the interferometer curve of any reflections from the walls must be very small.

I was not able to detect any well-marked effect of the parabolic mirrors upon the wave-length of the senders. There is, as already stated, a very marked accentuation of the maximum to the left of the central maximum, which I think must be due in some way to the action of the mirrors, since it does not occur when they are not used. This is particularly well shown in Figs. 10 and 11. It seems to me that when parabolic mirrors are used to concentrate the energy or to obtain a plane wave, it is important that their focal distance should be such that the sender and receiver will be in the neighborhood of *loops* of electric force, produced by reflection from its surface, and not in that of nodes. If this condition is observed, I do not believe that the measured wave-length will be affected by the presence of the mirrors. It is however conceivable, as I have already suggested earlier in this paper, that a parabolic cavity may have a natural period of its own, just as a hollow metal tube has been found to have a characteristic wave-length, and that that period might tend to mask that of the sender or receiver. I have not detected any such action, but the subject is, I think, worthy of further investigation.

I was unable to detect any effect of the length of the auxiliary spark-gaps upon the wave-length of the sender, at any rate within very wide limits. There seems to be a certain length which gives maximum excitation in the receiver, and in my experiments I found this to be about 2 mm.

One of the advantages of using a very high-resistance receiver is that it avoids the necessity of tuning. The method that I have employed in these experiments, and which could be followed generally, is to take a receiver of the approximate wave-length of the sender, and of two lengths less and greater. Five interferometer curves will be obtained. If the receiver is dead-beat, the wave-length will be the same for all. If not quite dead-beat the mean value of the wave-length determined from the five curves must be very close to the wave-length of the sender.

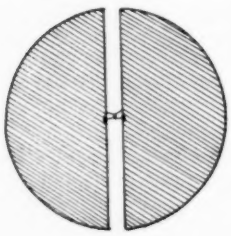

Fig. 13.
Disk-shaped receiver.

A receiver of unusual form was constructed when I was seeking to find a receiver that should be non-radiative. It was made of the

halves of a disk of copper foil (see Fig. 13) of 5 cm. radius, supported on the usual wooden holder. The wires of the thermal junction were soldered to the half discs, crossed and led back through the holder. It was placed in a vertical plane, with the slit perpendicular to the axis of the sender. Used with the 5-cm. sender, it was found to be very insensitive, and did not give any well-marked maxima or minima in the interferometer curve. I am inclined to think that it has no well-marked period of its own, and that it is a very poor radiator and absorber of electrical energy.

Webb and Woodman[1] have recently attempted to obtain an aperiodic linear receiver by using a receiver which is long compared with the wave-length measured. It appears to me that it is better to use a high-resistance receiver, such as I have described, first, because we have no guarantee that the long low-resistance received will not vibrate in overtones, and, second, because it is more convenient to handle.

In Figs. 14 and 15 are given the curves obtained with the high-

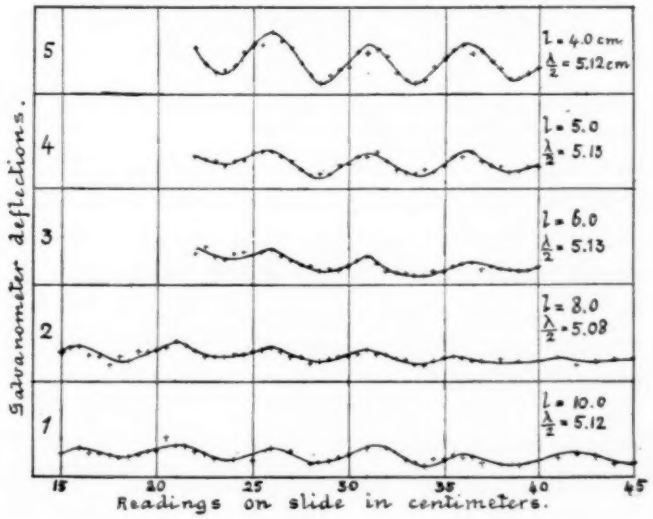

Fig. 14.

Interferometer curves for the 5-cm. sender obtained with high resistance receivers of iron wire of different lengths.

resistance receivers made of *iron* wire, .0015 cm. in diameter for the 5- and 7.5-cm. senders.

[1] *Loc. cit.*

They are similar to those obtained with the *constantan* wire receivers, but are less regular in form, probably for the reason suggested on p. 191. All the curves of Fig. 14 are in close agreement among

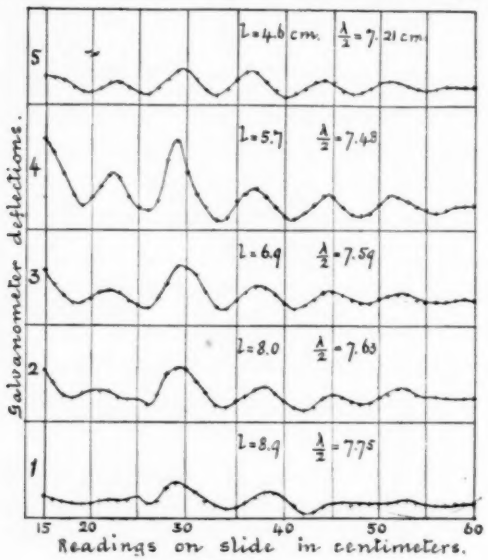

Fig. 15.

Interferometer curves for the 7.5-cm. sender obtained with high resistance receivers of iron wire of different lengths.

themselves, in giving $\lambda/2$ for the 5-cm. sender equal to 5.12 cm. In fact, so far as the wave-length is concerned, this value seems to me to be more probable than that of 5.21 cm. given by the constantan wire receivers. This would give the ratio of wave-length to length of oscillator, for the 5-cm. sender, equal to 2.08. It also only differs by about one per cent. from the value 5.06 cm. calculated from Abraham's formula.

SUMMARY.

1. The wave-length of the linear oscillator is slightly greater than twice its length, and within the limits of accuracy of this experiment is given by Abraham's formula. The ratio of wave-length to length of oscillator was found to be 2.03, 2.04, 2.08 for the 10-, 7.5- and 5-cm. oscillators, respectively.

2. The logarithmic decrement was found to be of the same order as that given by Abraham's formula. For the same length of spark-gap the total decrement decreased as the length of the oscillator increased.

3. The dependence of the apparent wave-length of an oscillator upon the wave-length of the receiver is due to the oscillation of the receiver itself. This difficulty may be overcome by using a receiver of very high resistance.

4. When the receiver is not dead-beat, the interferometer curve is the sum of two damped harmonic curves, one having half the wave-length of the sender, and the other half the wave-length of the receiver.

5. The length of the spark-gap of a linear oscillator has very little effect upon its wave-length.

6. When parabolic mirrors are used of such a focal length that the oscillator or receiver does not lie in a node of electric force, they do not appear to affect the wave-length of the oscillator.

7. Within the limits of accuracy of the experiments the wave-length of an oscillator 10 cm. long, supported by the hard-rubber cup containing kerosene oil, is the same as when free in air; that is, its wave-length is not sensibly affected either by the presence of the hard-rubber cup, or the kerosene oil, or by both together.

8. The direct wave from the sender does not sensibly affect the interferometer curve.

9. The lengths of the auxiliary sparks, within wide limits, do not affect the wave-length of the sender.

10. The use of a high-resistance receiver does away with the necessity of tuning.

11. An approximate theory for the high-resistance linear oscillator, and the general theory of the interferometer for damped electric waves, is given.

UNIVERSITY OF CINCINNATI,
February, 1910.

THE NATURE OF SPARK DISCHARGE AT VERY SMALL DISTANCES.

BY ELMER H. WILLIAMS.

HISTORICAL.

THE general subject of spark discharge in gases has commanded the attention of investigators for many years, dating back to the time of Volta.[1] However, the earlier experiments of Volta, Riess[2] and others of their time possess little more than historic interest, since the means at their disposal for measuring differences of potential was unreliable. It was not until 1860 that Lord Kelvin,[3] then Professor William Thomson, conducted the first trustworthy investigation on the spark discharge between two electrodes and the potential difference necessary to produce discharge with varying distances. In his researches, Kelvin measured the distances by which the plates were separated by means of a micrometer screw, and the potential differences necessary to produce a discharge across the intervening dielectric were measured with the absolute electrometer. The results obtained show that the difference of potential necessary to break down the intervening medium is not directly proportional to the distance between the electrodes. Later, important investigations were carried on by Baille,[4] Liebig,[5] Paschen[6] and Peace,[7] in which they showed that, for pressures considerably greater than the "critical pressure" and for comparatively large distances, the relation between the spark potential and the spark-length is a linear one, *i. e.*, if V is the spark potential measured in electrostatic units and x the spark-length at atmospheric pressure, measured in centimeters, then

$$V = ax + b,$$

[1]Volta, Identita, p. 53; Earhart, Phil. Mag. (6), 1, p. 147.
[2]Riess, Pogg. Ann., Vol. 40, p. 333.
[3]Lord Kelvin, Collected Papers on Electrostatics and Magnetism, p. 247.
[4]Baille, Annales de Chimie et de Physique (5), 25, p. 486, 1882.
[5]Liebig, Phil. Mag. (5), 24, p. 106, 1887.
[6]Paschen, Wied. Ann., 37, p. 79, 1889.
[7]Peace, Proc. Roy. Soc., 52, p. 99, 1892.

where a and b are constants depending on the gas. Paschen established the law that, for given potential differences, the product of the sparking distance and the gaseous pressure for producing a spark is a constant, *i. e.*, if d is the spark length and p the pressure of the gas, V is a function of pd. Peace, using parallel plate electrodes, found that there was a minimum spark potential, *i. e.*, if the pressure is kept constant and the length varied there is a point, depending on the pressure of the gas, at which the potential increases with the decrease of distance (see Fig. 1); also, that if the distance

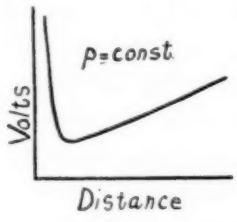

Fig. 1.

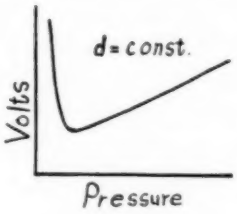

Fig. 2.

is kept constant, and the pressure varied the same phenomenon is observed (see Fig. 2). Peace found this point, called the minimum spark potential, to be independent of the spark-length. However, he did not investigate the phenomenon for very small distances. In air, the minimum spark potential is given as about 350 volts.

The developments of the electron theory have brought the subject of spark discharge into especial prominence within the last decade. Important investigations have been made by Earhart,[1] Carr,[2] Kinsley,[3] Hobbs[4] and Almy,[5] in which they have used refined methods for measuring the spark-lengths. By means of the interferometer they have been able to measure accurately distances to one twentieth of a wave-length of sodium light. Due to the fact that the above investigators used spherical electrodes, their curves do not show the increase of potential difference with diminishing distance between the electrodes that Peace obtained in his observations, since the least distance between the electrodes is not necessarily

[1]Earhart, Phil. Mag. (6), 1, p. 147, 1901.
[2]Carr, Proc. Roy. Soc., 71, p. 374, 1903.
[3]Kinsley, Phil. Mag. (6), 9, p. 692, 1905.
[4]Hobbs, Phil. Mag. (6), 10, p. 617, 1905.
[5]Almy, Phil. Mag. (6), 16, p. 456, 1908.

equal to the spark-length. Thus, when the distance is less than the critical spark-length, the spark will pass, not across the shortest distance, but across the place where the distance is equal to the critical spark-length. Earhart carried his measurements to very short distances and found that when the distance between the electrodes falls to less than about 3×10^{-4} cm., there is a very rapid diminution of the spark potential which, according to his results, seems to become directly proportional to the distance.

The results of Kinsley and Hobbs agree with those of Earhart in that they show a rapid diminution of the spark potential for very short distances. Kinsley, by a special arrangement of his interferometer, was able to measure exceedingly short distances; in some cases he records spark-lengths as small as 3×10^{-7} cm., and a sparking potential of only one volt. Hobbs obtained the exceedingly important and suggestive result that for spark-lengths below about 3×10^{-4} cm., the relation between the spark-length and the potential difference is independent of the pressure and nature of the gas, while it does depend upon the nature of the metal of which the electrodes are made.

If the results of Earhart, Kinsley and Hobbs are correct, Paschen's law does not hold for very short distances, and the phenomenon of discharge at very short spark-lengths must be different from that at greater distances. Now, J. J. Thomson has shown that the discharge for comparatively large distances is due to ionization by collision of the gas between the electrodes.[1] If, for very small distances, ionization does not take place to such an extent as to make the gas conducting, the discharge must take place by some other means. It might be carried by a current of electrons, by a current of positive ions, by currents of both electrons and positive ions, or by material particles torn from the electrodes themselves. In any of these cases we should expect that the material of which the electrodes are made would exert a large influence upon the discharge potential for any given distance.

J. J. Thomson[2] offers a "possible explanation" of the behavior of the discharge when the electrodes are very close together. He

[1] J. J. Thomson, Conduction of Electricity through Gases, p. 270.

[2] J. J. Thomson, Conduction of Electricity through Gases, p. 456.

makes use of the hypothesis that in a metal, even at ordinary temperatures, free corpuscles are moving about in every direction. Now, if these corpuscles could escape from the metal under ordinary conditions, the metal would be unable to retain a charge of negative electricity. J. J. Thomson states that one of the reasons the corpuscles do not escape is that as soon as they leave the metal there is an electrostatic attraction between the corpuscle and the metal which drags the corpuscle back. Assuming the external electric force which Earhart obtained in his experiment, he finds, by a simple calculation, that if the distance of the corpuscle from the surface of the metal exceeds a certain amount, 10^{-7} cm., the pull of the external field would be able to drag the corpuscle away, thus causing that electrode to act like a cathode. This discharge would, therefore, have the character of a small arc or of the hot lime cathode, and a discharge of negative electricity would pass to the opposite electrode. J. J. Thomson further states that if his explanation is correct "the discharge across these very small distances is entirely carried by the corpuscles and no part of it by the positive ions."

Referring to the observation of Hobbs, that the spark potential depends on the material of which the electrodes are made, the above author suggests that "it would be interesting to see whether these carriers are corpuscles or a mixture of negatively electrified corpuscles and positively electrified atoms." "This could be tested," he says, "by making the two electrodes of different metals; if all the carriers are corpuscles the potential difference would depend only upon the metal used for the negative electrode; if they were all positively electrified atoms, then only the positive electrode would be active, while if they were a mixture both electrodes would affect the potential."

Earhart,[1] in 1908, made a series of observations with the object of determining if possible, the nature of the carriers of the discharge. He used the three metals platinum, silver and aluminium, and from his observations came to the conclusion that both positive ions and negative corpuscles are the carriers for the discharge produced by potential differences less than the least ionizing potential. Earhart,

[1]Earhart, Phil. Mag. (6), 16, p. 48, 1908.

however, made only a small number of observations with each pair of electrodes and his readings varied widely for each spark distance, so that it does not appear that he is fully justified in his conclusions.

Later Almy,[1] investigating the subject of spark potential, came to a conclusion that is entirely different from that of Earhart, Kinsley and Hobbs. He states that his evidence seems conclusive that with spark-gaps down to at least 0.3 wave-length of sodium light a potential of 330 volts is not sufficient to produce discharge through air at atmospheric pressure. Almy used very small platinum spheres, 0.007 cm. in diameter, and also needle points, and he suggests that the electrostatic force which exists between two large electrodes brought within such short distances as were used by Earhart, Kinsley and Hobbs, is sufficient to bring the electrodes into contact at potentials less than the "minimum," so that the potentials that produced short circuits in the spark-gaps were simply those required to give the requisite displacement of the electrodes. If this be true, it is difficult to understand why Hobbs obtained the result that the spark discharge depends on the material of the electrodes and not on the nature of the gas in such a regular way.

Since this question has a very important bearing in the electron theory, the present investigation was undertaken not only to clear up the apparent discrepancies, but also to investigate the whole question of electric discharge at short distances.

DESCRIPTION OF APPARATUS.

The apparatus consisted of electrodes mounted on a standard Michelson interferometer made by Gaertner. The whole was inclosed in an air-tight iron box. The movable carriage of the interferometer *C*, Fig. 3, carried the plane electrode *E*, a disk about 2 cm. in diameter which was fitted carefully into the socket *S* and held in place by the nut *N*. A part of the socket *S* which was of greater diameter, was fitted to receive a worm screw *W*, by which the electrode could be turned about a longitudinal axis, thus presenting a new surface before each discharge. The shaft, on the end of which was the worm screw, was arranged so that it could be disconnected after the disk had been turned, in order that the movable

[1] Almy, Phil. Mag. (6), 16, p. 456, 1908.

carriage might be free from any vibrations or distortions of the sides of the iron box. The pressure with which the socket *S* fitted against

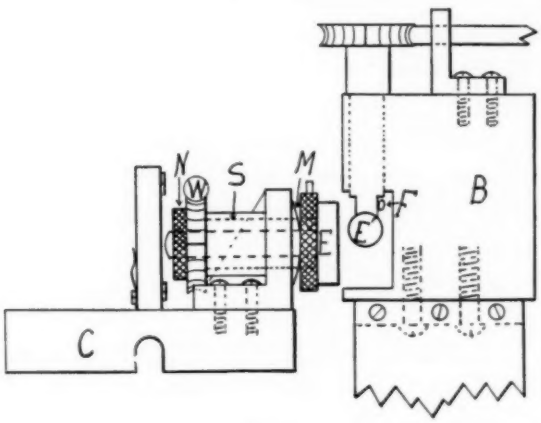

Fig. 3.

its bearings was regulated by the spring washer *M*. This was made as great as possible, the limit being that which could be overcome by the worm screw *W*.

The support of the spherical electrode was attached to the interferometer bed itself in order to avoid vibrations and to make the electrode more rigid. In the original form, this support consisted of a heavy, vertical brass plate through which passed, at an angle of about 45°, the rod carrying the ball. The bearings were carefully ground cones held in place by a heavy spring. However, it was found that this could not be made rigid enough to withstand the electrostatic forces without being pulled to the other electrode. After several changes, the arrangement shown in Fig. 3 was adopted. This consisted of a heavy brass block *B*, which could be rigidly clamped to the interferometer base. A vertical hole was drilled into this block and a section of the block removed. Into this hole was inserted a closely fitting rod carrying at its lower end the ball electrode *E'*. Although the cylinder and rod of this electrode fitted perfectly so far as could be detected, yet to guard against a possible movement of the magnitude of one wave-length of light or less, a spring *F* was placed behind the rod so that it was always held against the front surface of its receptacle. In order to take up lost motion

of the movable carriage and to facilitate manipulation, heavy rubber bands were placed around the supports of the electrodes close to their bases. This device also gave greater stability.

In Fig. 4 is shown a diagram of the electrical connections. Circuit *a* consists of a galvanometer, an E.M.F. of about one volt, and the electrodes in series. The deflection of the galvanometer indicated when the electrodes were in contact, also the instant at which contact was broken. The breaking of the circuit between the electrodes could be detected to within one tenth of a fringe, or one twentieth of a wave-length of sodium light. Circuit *b* contained the source of E.M.F., the electrodes and a high resistance *R* in series. The E.M.F. was furnished by a bank of small storage cells *B*, which were connected across a water rheostat. The plunger *P* could be raised vertically, thus increasing the fall of potential between its lower end and the bottom of the tube *T*. A movement of 0.5 cm. corresponded to a change of one volt between the electrodes. A Kelvin electrostatic voltmeter, in shunt with the electrodes, was used to measure the difference of potential. This was calibrated frequently by means of a potentiometer and standard cell, and was found to be very constant. The results were also checked from time to time with a Weston standard "semi-portable" voltmeter.

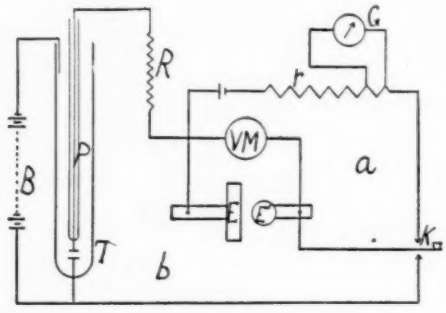

Fig. 4.

METHOD AND PROCEDURE.

The surfaces of the electrodes were carefully polished with fine emery powder and jewelers' rouge and placed in position. The box was then sealed, the air replaced by dry, dust-free air, and allowed to stand for some time. On beginning the observations, the electrodes were brought into contact and separated again by means of the micrometer screw, the point at which the surfaces ceased to be in contact being indicated by the galvanometer *G*. After the circuit

had been broken between the electrodes, they were separated until a number of fringes n had passed the cross-hair of the telescope that was focused on them. Then $n/2$ equals the distance in wave-lengths between the plates, since the passage of a fringe corresponds to one half wave-length displacement of the carriage.[1] The main circuit b was then closed and the plunger p, of the water rheostat, raised very slowly until the discharge potential was reached. During the latter part of this increase in potential, stops of from one to two minutes were made after each increase of one volt, in order that the effect of lag might be eliminated.

Electrostatic Force Between the Electrodes.

The electrostatic force between the electrodes is a very important factor in the determination of spark potentials at very short distances. In order to gain an idea of the magnitude of this force, the following calculation was made. Experimental tests were also carried on for the same purpose.

The electrostatic force between two electrodes, one a plane and the other a sphere of 1 cm. diameter, where the electrodes are any given distance apart, say one wave-length of sodium light, may be calculated, approximately, in the following way.

Divide the plane A, Fig. 5, into areas whose boundaries are distant 1λ, 2λ, 3λ, etc., from the sphere B. Call these areas A_1, A_2, A_3, etc. We can consider any one of these areas, A_n, to be surrounded by a guard ring and compute the attraction between it and an equal area A_n', whose distance from A_n is equal to the shortest distance between A_n and the corresponding area of the sphere. This force between A_n and A_n' will be greater than the force between A_n and the above mentioned area of the sphere. Thus, by taking the sum of the forces on all the areas, we will obtain an upper limit of the attraction between the two electrodes.

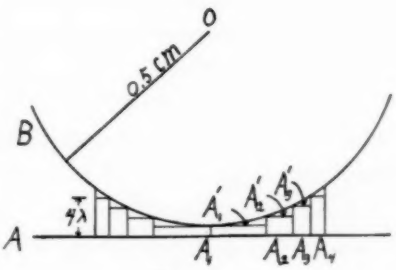

Fig. 5.

[1] Mann, Manual of Advanced Optics.

A simple geometric consideration leads to the following result:

$$A_n = \text{const.}$$
$$= 1.87 \times 10^{-4} \text{ sq. cm.}$$

The force acting on the plane A is

$$F = \sum_1^n F_{A_x}.$$

Now this force will be less than the force between the plane A and the planes A_1', A_2', etc., of Fig. 5.

Therefore,

$$F < 1.87 \times 10^{-4} \sum_1^n F'_{A_x},$$

where F'_{A_x} equals the force per unit area between any area A_x and the corresponding area A_x'. This force may be calculated from the general formula,

$$F_{gr.} = \frac{V^2 A}{8\pi d^2 g}.$$

If we assume $V = 300$ volts, *i. e.*, one electrostatic unit of potential, and $n = 20$, we get

$$F < 3.41 \text{ gr.}$$

When $n = 20$, the force per unit area is only one fourth of one per cent. of what it is when $n = 1$, and since the series is convergent, the terms after this may be neglected without introducing any appreciable error.

Maxwell[1] gives the following formula for the electrostatic force between two spheres whose radii are a and b, and the distance between whose centers is c.

$$F = \frac{1}{2}\left[V_a^2 \frac{dq_{aa}}{dc} 2V_a V_b \frac{dq_{ab}}{dc} + V_b^2 \frac{dq_{bb}}{dc} \right].$$

If the radius of one is taken equal to one centimeter and that of the other very great, and the distance between the electrodes is taken to be of the magnitude of one wave-length of light, the formula converges so badly that it cannot be easily applied.

[1]Maxwell, Electricity and Magnetism, Vol. I., p. 272.

Experimental Test of Stability.

The preliminary work of this investigation consisted of testing the stability of the electrodes. The electrode attached to the movable carriage was chosen upon which to experiment as it was the one to which the method of attack was most easily applied. It was tested in the following way. The mirror was removed from the carriage and a plane electrode, polished so that it would give good fringes, was mounted in its place. A spherical electrode 3 or 4 mm. in diameter was placed in front of this plane and rigidly clamped to the interferometer base. The carriage of the interferometer was loaded with extra weights placed centrally over the bearings. The electrodes were then brought into contact and separated again to distances of 1λ, 2λ, 3λ, etc., after which the voltage was applied. Repeated observations with voltages as high as 300 and spark-gaps of 1λ and 2λ showed no perceptible movement of the electrode. A movement of one fourth wave-length, or one half fringe, could easily have been detected. This showed that it was possible to construct electrodes that could withstand the electrostatic forces which exist between them without being drawn together. However, this electrode had no arrangement whereby a new surface could be presented from without an air-tight box. Later, when this arrangement was added, great difficulty was experienced in getting a construction by which the electrode was free to turn and yet retain the necessary stability. The form described on the previous pages was finally adopted. This form was not tested by the above sensitive method, since the nature of the data obtained indicated quite accurately the degree of stability.

Results.

1. *Effect of Kind of Material of Which the Electrodes Are Made upon the Discharge Potential.*—Since the primary object of this research was to investigate the effect of the material of the electrodes upon the spark potential with a view of determining therefrom the nature of the discharge, a series of observations was made with the materials platinum, silver, aluminium and brass in a number of combinations. Tables I. to IX. inclusive give the results of these observations. Distances in wave-lengths are given in the column

under λ and potential differences in volts in the column under V. Each table includes from two to three series taken on different days.

TABLE I.

Al+. *Al*−.

λ	V	λ	V	λ	V
1	323	2	343	3	370
1	298	2	370	3	369
1	284	2	369	3	370
1	310	2	371	5	370
1	263	2	370	5	371
1.5	357	2	370	5	371
1.5	371	3	372	5	370
1.5	368	3	359	5	372
1.5	347	3	371	5	368
1.5	370	3	371	5	371

TABLE II.

Brass+. *Al*−.

λ	V	λ	V	λ	V
1	302	1.5	300	3	369
1	372	1.5	341	3	371
1	277	1.5	372	3	368
1	258	1.5	371	3	370
1	269	1.5	369	3	370
1	347	2	346	3	366
1	370	2	370	3	371
1	371	2	371	5	369
1.5	371	2	369	5	371
1.5	369	2	339	5	371
1.5	326	2	370	5	368
1.5	370	2	350	5	369
1.5	372	2	369	5	371
1.5	348	2	370	5	370
1.5	370	3	369	5	371

Table III.

Brass −. *Al* +.

λ	*V*	λ	*V*	λ	*V*
1	371	1.5	340	3	346
1	337	1.5	371	3	370
1	324	2	325	3	369
1	346	2	370	3	370
1	369	2	369	3	371
1	369	2	357	5	363
1	289	2	371	5	369
1.5	371	2	333	5	371
1.5	310	2	371	5	371
1.5	368	2	316	5	370
1.5	370	3	368	5	369
1.5	369	3	371	5	370
1.5	354	3	370	5	372
		3	370	5	368

Table IV.

Pt +. *Pt* −.

λ	*V*	λ	*V*	λ	*V*
1	357	1.5	340	3	371
1	345	1.5	371	3	369
1	370	1.5	370	3	370
1	335	2	370	3	371
1	319	2	371	5	372
1	284	2	367	5	370
1	332	2	370	5	371
1.5	371	2	371	5	369
1.5	369	3	369	5	370

Table V.

Al +. *Ag* −.

λ	*V*	λ	*V*	λ	*V*
1	323	1.5	369	3	369
1	316	1.5	346	3	371
1	368	2	372	3	372
1	334	2	371	3	371
1.5	372	2	370	5	370
1.5	370	2	370	5	371
				5	371

Table VI.

Pt+. *Ag*−.

λ	*V*	λ	*V*	λ	*V*
1	241	1.5	372	3	371
1	306	1.5	370	3	372
1	331	1.5	371	3	370
1	368	2	371	5	369
1	346	2	372	5	371
1.5	372	2	372	5	372
1.5	317	2	371		

Table VII.

Brass+. *Pt*−.

λ	*V*	λ	*V*	λ	*V*
1	343	1.5	330	3	372
1	324	1.5	368	3	369
1	286	1.5	370	3	371
1	331	1.5	368	3	364
1	304	2	368	3	370
1	345	2	372	3	368
1	299	2	369	3	369
1	318	2	370	5	370
1.5	366	2	369	5	372
1.5	367	2	368	5	368
1.5	368	2	370	5	370
1.5	370	2	343	5	368
1.5	367	2	369	5	371
1.5	364	3	371	5	370

TABLE VIII.

Brass −. *Pt* +.

λ	*V*	λ	*V*	λ	*V*
1	340	1.5	306	3	371
1	262	1.5	371	3	372
1	356	2	371	3	372
1	320	2	370	3	370
1	344	2	368	3	371
1	365	2	370	5	372
1	337	2	372	5	372
1.5	372	2	367	5	372
1.5	371	2	371	5	371
1.5	372	2	370	5	372
1.5	370	2	372	5	371
1.5	372	3	372	5	371
1.5	367	3	372	5	372
1.5	370	3	369		

TABLE IX.

Al −. *Ag* +.

λ	*V*	λ	*V*	λ	*V*
1	255	1.5	370	3	371
1	290	1.5	338	3	372
1	321	2	372	3	372
1	273	2	372	3	372
1.5	371	2	371	5	372
1.5	304	2	370	5	372
1.5	327	2	372	5	371

The results given in these tables show beyond a doubt that the material of which the electrodes are made has no effect on the discharge potential. It is true that in those cases where aluminium is used the values for 1λ, and even for 1.5λ and 2λ, are not as consistent as in the case of other metals. This, however, is due to a mechanical difficulty. Aluminium is by far the softest metal of the above list and when attempts were made to polish it, instead of the projection being removed they were pushed into the depressions. The result was that a very good polish could be obtained yet when the alumin-

ium electrode was subjected to strong electrostatic forces, with one of the original depressions opposite the other electrode, the material filling the depression was pulled out producing a metallic bridge and thus short circuit.

In these researches, I have not attempted to work with distances below one wave-length of sodium light. There are two reasons for this: (1) The inability to get perfectly stable electrodes, which make readings for shorter distances unreliable; (2) the limit to the degree of polish which could be obtained with the means at hand.

Not only do the above tables show that the discharge potential does not depend upon the material of which the electrodes are made, but also that there is no decrease in the discharge potential for distances below five wave-lengths of sodium light. In Fig. 6 is

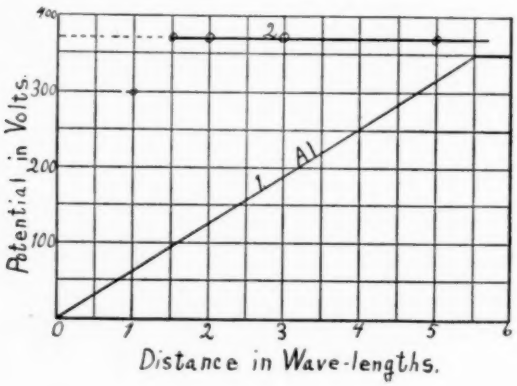

Fig. 6.

shown a representation of the results which I obtained as given in Table I. (curve no. 2), and also the curve for aluminium electrodes given by Hobbs in his paper (curve no. 1). Whereas the knee in the curve given by Hobbs occurs at 5.6λ, the same does not occur in my results until the distance between the electrodes is as short as 1.5λ or less. Since the conditions are the same with the exception of the mechanical construction, we must conclude that if we could construct perfectly stable electrodes and produce a perfect polish, the knee of the curve could be made to approach the y-axis to within at least a small fractional part of a wave-length of light. It has been calculated that an electric force of about 10^9 volts per cm. is

sufficient to drag an electron out of an atom. Now, a difference of potential of 370 volts across a distance of .005 wave-length would give this force, so that within this distance the discharge might have the character of a small arc of of a hot lime cathode as suggested in the theory of J. J. Thomson.

2. *Path of the Discharge.*—Since, at atmospheric pressure, the discharge potential remains constant for distances below about 10λ, the path length of the discharge must be constant, *i. e.*, if one plane and one spherical electrode is used, as in my case, the discharge

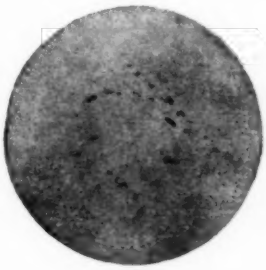

Fig. 7.

Fig. 8.

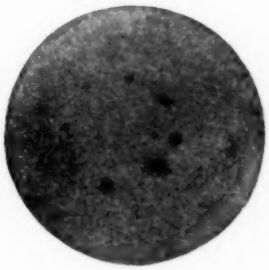

Fig. 9.

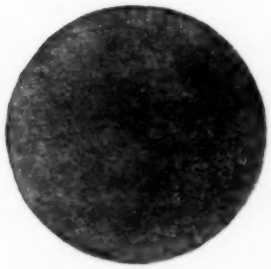

Fig. 10.

path will not be across the shortest distance. To verify this I have taken photo-micrographs of the plane electrode at the point where discharge occurs (see Figs. 7 to 10). The magnification of the photo-micrographs is 50. In Fig. 9, a capacity of .01 M.F. was shunted across the electrodes so that a thicker spark and larger pit was produced. In the other photographs no capacity was used. When the distance between the electrodes is 2λ (Figs. 7 and 8) and 3λ (Fig. 9), the discharge produces on the polished surface of the

plane electrode a circle with the center entirely free from pits. Examination of the sphere showed also a circle of pits, but in the latter case it was impossible to photograph the circle because of the inability of getting large enough area of the spherical electrode within the focus of the photo-micrographic apparatus. In Fig. 10 the distance between the electrodes is 5λ and we see that the circle becomes much smaller—the space within which there are no pits having almost disappeared. Thus for distances between the electrodes equal to 2λ and 3λ it is evident that the discharge does not take place across the shortest distance, but across a place where the distance is equal to the critical distance for the pressure involved. According to the experiments of Hittorf[1] and others, we should expect that if the pressure were reduced the diameter of the circle would become greater. From the fact that each spark disturbs the metal of the electrodes, we should expect that the light of the spark should contain line spectra of the metals between which the sparks occur.

3. *Effect of Ionization by Means of Ultra-violet Light.*—I have shown that the nature of the material of which the electrodes are made has no effect upon the discharge potential, also, that the path of the discharge is not the shortest path between the electrodes when these are very close together; therefore the nature of the discharge must be the same for very short as for greater distances; *i. e.*, ionization is the important factor. If the number of ions per cubic centimeter is increased, the chances of ionization due to collision of the ions moving under the influence of the strong electric field is increased proportionally, and the spark will pass at a lower potential. In my investigations, the gas surrounding the electrodes was ionized by means of ultra-violet light for some time, in most cases from eight to ten minutes, and readings were taken the same as before. A comparison of these results, given in Tables X. to XV. with the results obtained with the same electrodes (see Tables VIII. and IX.) when no ionizing agent was present other than the electrostatic force which existed between the electrodes, substantiates the above conclusions.

[1]Hittorf, Wied. Ann., 21, p. 96, 1884.

TABLE X.

Br.−. *Pt*+.

λ	*V*
1.5	145
1.5	170
1.5	150
2	170
2	170
2	160
3	260
3	230
5	320
5	320

TABLE XI.

Br−. *Pt*+.

λ	*V*
1.5	160
2	170
2	150
2	150
2	150
3	240
3	220
5	315
5	315

TABLE XII.

Br−. *Pt*+.

λ	*V*
1.5	140
1.5	150
1.5	120
1.5	150
2	160
2	160
2	150
3	260
3	220
5	300
5	310

TABLE XIII.

Al−. *Ag*+.

λ	*V*
2	110
2	180
2	150
5	315
5	320
5	320

TABLE XIV.

Al−. *Ag*+.

λ	*V*
2	150
2	180
2	160
2	150
3	230
5	310
5	315
5	320

TABLE XV.

Al−. *Ag*+.

λ	*V*
2	190
2	160
2	160
2	150
3	210
3	260
3	240
5	310
5	310
5	330

It is interesting to note that while the electrostatic force which exists between the electrodes, which is about 5×10^6 volts per cm. when the distance is 1λ and the voltage 300, is unable to produce ionization, the force contained in a beam of light *is* able to do so. This is a strong point in favor of the electromagnetic emission theory of light as advocated by J. J. Thomson[1] and Jakob Kunz.[2]

Ionization consists in dragging an electron out of an atom or molecule. At ordinary temperatures this soon combines with another atom so that a negative ion is an atom that has acquired an electron, and a positive ion is an atom that has lost an electron.

[1]J. J. Thomson, Phil. Mag. (6), 19, p. 301, 1910.

[2]Jakob Kunz, PHYS. REV. 29, p. 212, 1909.

If the ionizing agent is a Roentgen pulse, the distribution of energy in the electromagnetic field is continuous according to Maxwell's theory of electromagnetism. If this is true and one of the atoms struck by the pulse is ionized, why are not all ionized? It cannot be due to a special property of the atoms themselves for then ionization would be affected by temperature which is not the case.

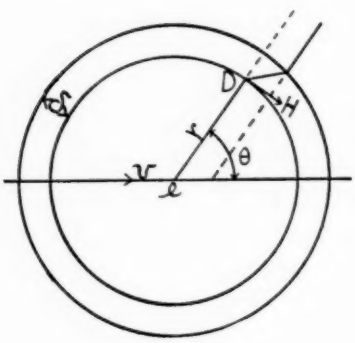

Fig. 11.

According to the theory that the distribution of energy is continuous, the tangential electric force E_t in the direction DH of Fig. 11 is

$$E_t = \frac{ev \sin \theta}{rc\delta}.$$

If

$$\begin{aligned} e &= 4.65 \times 10^{-10}, \\ \sin \theta &= 1, \\ r &= 10, \\ v &= 10^{10}, \\ \delta &= 10^{-8}, \\ c &= 3 \times 10^{10}; \end{aligned}$$

then

$$E_t = .46 \text{ volt per cm.}$$

The force with which an electron is attached to an atom may be obtained from optical phenomena. Neglecting damping by radiation, the equation of motion of an electron in an atom may be written in the form

$$m\frac{d^2x}{dt^2} = -X.$$

Let

$$X = a^2x = E_re,$$

or

$$a^2 = E_re/x,$$

where x is the displacement and E_r the restoring force.

The time of vibration of the electron is

$$T = 2\pi\sqrt{\frac{m}{a^2}}$$

$$= 2\pi\sqrt{\frac{mx}{E_r e}},$$

or

$$T^2 = \frac{4\pi^2 mx}{E_r e}.$$

therefore

$$E_r = \frac{4\pi^2 mx}{T^2 e}.$$

Take x = radius of atom = 10^{-8} cm. (approx.),

$$m = 8.8 \times 10^{-28} \text{ gr.},$$
$$e = 4.65 \times 10^{-10}.$$

If we assume that the electron vibrates with the frequency of sodium light

$$T = 2 \times 10^{-15}.$$

Then

$$E_r = 2 \times 10^5 \text{ absolute electrostatic units per cm.}$$
$$= 6 \times 10^7 \text{ volts per cm.}$$

We may also estimate the order of magnitude of E_r by considering the energy required to produce an ion, *i. e.*, $U = 4.4 \times 10^{-11}$ ergs, and writing this energy as

$$U = \int_0^x X dx = \int_0^x a^2 x dx$$

$$= \frac{a^2 x^2}{2}$$

$$= \frac{E_r e x}{2};$$

therefore

$$E_r = \frac{2U}{ex}.$$

The values of U and e are known; therefore we have only to assume the value of x, the radius of the atom, whereas in the previous

consideration we had also to assume T, the period of vibration of the electron. Assuming x to be 10^{-8} cm. we have

$$E_r = 2 \times 10^7 \text{ absolute electrostatic units per cm.}$$
$$= 6 \times 10^9 \text{ volts per cm.}$$

This is 10^{10} times greater than the electrical force in a Roentgen pulse.

Since the electrical force of the Roentgen pulse is so small compared with the restoring force acting on the electron, it is surprising that any atoms are ionized at all.

A large number of pulses might, if they came regularly, ionize an atom, but in the case of Roentgen pulses there is no regularity, *i. e.*, resonance is impossible.

Let the ionizing agent be ultra-violet light. Assuming the wave theory, how long a time will be required to produce an ion? Let us assume that the maximum electric force E_0 of Fig. 12, acts all the time and that the force acts always in the same direction; also that there is no loss due to radiation, *i. e.*, no damping. Each of these assumptions will tend to make the time less than in the actual case. The velocity imparted to an electron during a half period is

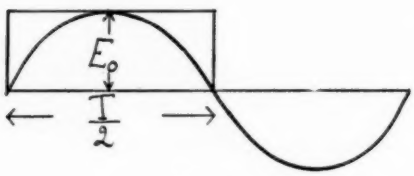

Fig. 12.

$$v = \frac{E_0 e}{m} \frac{T}{2}.$$

The kinetic energy acquired by the electron during this time will be

$$\text{K.E.} = \tfrac{1}{2}mv^2 = \tfrac{1}{8}\frac{E_0^2 e^2 T^2}{m},$$

and in one second it will acquire

$$\text{K.E.}_1 = \tfrac{1}{4}\frac{E_0^2 e^2 T}{m}.$$

Assuming the maximum electrical force E_0 of ultra-violet light to

be 0.5 volt per cm., which is probably high, we have

$$E_0 = 1/600,$$
$$e = 4.65 \times 10^{-10},$$
$$T = 10^{-15},$$
$$m = 8.8 \times 10^{-28};$$

therefore

$$\text{K.E.}_1 = 1.7 \times 10^{-13} \text{ ergs.}$$

The probable value of the energy required to drag an electron out of an atom is 4.4×10^{-11} ergs. Therefore, on the above assumptions, the time required for ultra-violet light to produce one electron would be

$$\frac{4.4 \times 10^{-11}}{1.7 \times 10^{-13}} \text{ sec.} = 258 \text{ sec.}$$
$$= 4.3 \text{ min.}$$

In my investigations with ultra-violet light, given in Tables X. to XV., the effect, on applying the light to the dielectric when the potential difference was many volts below the discharge potential, was *instantaneous*. For example, while the discharge potential for a distance of two wave-lengths is 370 volts for dry air at atmospheric pressure, yet, if ultra-violet light was applied when the potential difference was as low as 250 or even 200 volts, a discharge took place instantly. In obtaining the data given in Tables X. to XV. the light was applied continuously. To begin with, a potential of 30 or 40 volts was applied; this was raised slowly until the potential was reached where discharge occurred which, in most cases, took from eight to ten minutes as stated above. Thus the wave theory cannot account for ionization by means of ultra-violet light, whereas the electromagnetic emission theory can.

It remains undetermined whether the light acts primarily on the metal, or on the molecules of the gas, or on both.

4. *Nature of the Gas: Effect of Moisture.*—Early in the course of my work some readings were taken with the electrodes surrounded by the air of the room; later the apparatus was placed in an iron box and surrounded by dry dust-free air. It was noticed that the discharge potential in the latter case was considerably higher than

in the former, although both conditions gave very consistent results. In order to determine more exactly the effect of moisture upon the

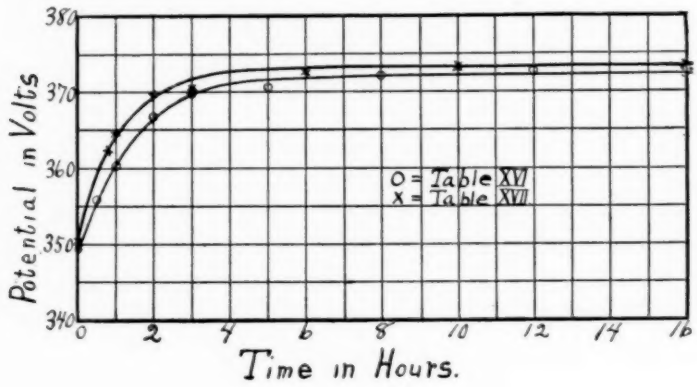

Fig. 13.

discharge potential, the apparatus was surrounded by air which had been passed through two tubes, one containing $CaCl_2$ and the other cotton-wool. The air was passed through at such a rate that

TABLE XVI. TABLE XVII.

Br −. *Pt* +. *Distance* = 5λ.

Time.	Voltage.	Time.	Voltage.
9:00 A. M.	349	11:00 A. M.	350
" "	350	" "	350
9:30 "	355	11:40 "	362
" "	357	" "	363
10:00 "	361	12:00 "	364
" "	360	1:00 P. M.	370
11:00 "	366	" "	369
" "	367	2:00 "	370
12:00 "	369	5:00 "	372
" "	370	" "	373
2:00 P. M.	371	9:00 "	373
" "	370	" "	373
5:00 "	372	8:00 A. M.	373
" "	372	" "	373
9:00 "	373		
9:00 A. M. (24 hrs.)	372		

only a part of the moisture was removed. Within the box containing the apparatus was placed a large vessel containing strong sulphuric acid. The change of the minimum potential with the degree of dryness of the air is shown in Tables XVI. and XVII., and graphically in Fig. 13. From these results it will be seen that the nature of the dielectric between the electrodes has an influence upon the discharge potential, *i. e.*, the presence of moisture lowers the discharge potential for very short as well as for greater distances. The results also indicate that for dry air at atmospheric pressure the minimum potential is 372 volts instead of 350 volts as usually given. It remains to be investigated how saturated air behaves.

Summary.

The following are the principal results which have been established by this investigation:

1. The material of which the electrodes are made has no effect upon the discharge potential.

2. The nature of the discharge for very short distances is the same as for greater distances.

3. The discharge potential for a distance between the electrodes of one wave-length of sodium light is the same as for five wave-lengths, being in both cases the minimum potential which is 372 volts.

4. When the distance between the electrodes is very short, 5λ or less, the path of the discharge is not along the shortest distance.

5. Ionization of the gas between the electrodes lowers the discharge potential.

6. The electric force in a beam of ultra-violet light is, according to the wave theory, 0.5 volt per cm.; yet the beam of light is able to produce an action which cannot be brought about by a constant electrostatic force of 5×10^6 volts per cm. This points to a modification of the wave theory as suggested by the electromagnetic emission theory of light.

7. The nature of the dielectric effects the discharge potential—the presence of moisture lowering this potential. For dry air the minimum potential is 372 volts whether the distance between the electrodes is 1λ or 5λ.

In conclusion, I desire to express my gratitude to Professor A. P. Carman for his kindly interest in my work, and to Professor Jakob Kunz for many valuable suggestions throughout the course of this investigation.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
April 15, 1910.

PHOTOGRAPHIC PHOTOMETRY AND SOME INTERESTING PHOTOGRAPHIC PHENOMENA.[1]

By Charles F. Brush.

PHOTOGRAPHIC photometry has grown to large importance during the last few years in connection with astronomical research, notably at Harvard Observatory. But it does not appear to have been developed far in other directions. That it may be usefully employed in some lines of physical research is beyond question.

During the last two years the writer has found this method of photometry most useful in a delicate investigation, and, incidentally, has developed the art in the direction of detecting and estimating small differences and changes in the transparency of bodies.

Fig. 1 is a diagram of the apparatus employed. An incandescent electric lamp, *h*, with heavily frosted bulb, is mounted in a well-ventilated box or lantern *i* adapted to be moved horizontally on a long bar *k*. Close to the surface of the lamp bulb is an iris diaphragm *l* controlling the diameter of a circular spot on the surface of the bulb, which spot is the source of light used in most of the experiments. A large thick wooden screen *n* has an opening *o* 5 mm. high and 19 mm. long, horizontally, with semicircular ends. A beam of light from *h* passes through the opening *o* and falls on a photographic plate *p* carried in a holder which may be quickly secured in any one of eighteen definite positions as shown at *t*, which is a front view of the plate. The opening *o* is closed normally by a drop-shutter *s* of thick sheet metal. A front view of the shutter, at the right, shows in dotted line the opening *o* behind it. The shutter moves with perfect freedom in vertical guides, and is retained in its upper position by a spring catch which may be released at pleasure. The slot in the shutter is of such length as to give a mean exposure of one ninth second. Of course

[1]Read before the American Association for the Advancement of Science, December 29, 1909.

the lower part of the spot of light through *o* is exposed less than the upper part, on account of the different speed of the shutter in opening and closing *o*. This difference is found by calculation to be about 2 per cent. It is corrected by vertical adjustment of the lamp. Experience shows that this shutter may be relied on to give constant exposures. A screen, not shown, covers the opening *o* while the shutter is being raised.

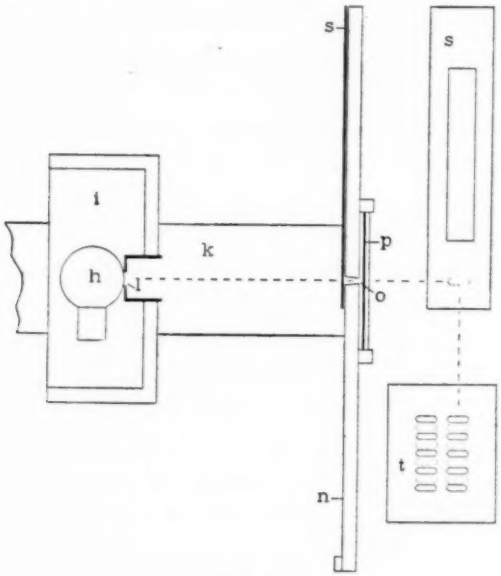

Fig. 1.

The lamp *h* is supplied with current from a large storage battery with no other load at the same time. Great care is taken to secure constant E.M.F. If the battery has been recently charged, it is slightly discharged for an hour or two with a light load, and allowed to rest several hours before using. Under these circumstances no change of E.M.F. can be detected during the progress of an experiment. All connections between the battery and lamp are soldered or tightly clamped, and the circuit is closed and opened with a heavy knife switch. Experiment has shown that the lamp attains full brilliancy within one minute after closing the switch, and does not change at all thereafter for at least four minutes. The lamp is lighted never less than one, nor more than two minutes before making an exposure.

The optical system, or body whose transparency is to be tested, is placed in the path of the beam of light from *l* to *o*.

The customary manner of using this apparatus is to expose one or two rows of five spots each on the photographic plate as shown in full lines at *t*, with four blank alternating spaces between. These spots are termed "comparison spots," and are used as a standard with which to compare the intermediate spots shown in dotted lines, and exposed subsequently, after the introduction, removal or change of the optical system in the path of the light.

In order to render this method of photometry as sensitive as possible, it is, of course, necessary to employ such plates, such exposure, and such development and other treatment, as will show in a print from the negative the greatest contrast with a given small change of light. The writer has experimented extensively with various plates and developers. A plate called by the maker "panchromatic process," is by far the best yet found. This has a thick coating of rather slow working emulsion sensitized for the red end of the spectrum. Tested in all parts of the spectrum it is found sensitive everywhere from the deep red far into the ultra-violet, except a narrow region in the green. Indeed, it is so sensitive to red and orange light that it must be handled and developed in total darkness. The latter is not difficult when the best time of development with a standard developer has been learned. By reason of this general sensitiveness the plate is two or three times as fast as the ordinary process plate.

The best developer thus far found is one of the standard hydrochinon developers, to which is added several times the normal amount of potassium bromide.[1] This developer, when made with pure chemicals, carefully weighed, and dissolved in proper sequence in accurately measured volumes of distilled water, may be relied on to give constant results. To secure even development of a 4″ x 5″ plate, it is found advisable to use a 7″ x 9″ developing tray with a sunken panel across the middle just large enough and deep enough to receive the plate and leave the film flush with the

[1]Distilled water 16 ounces, desiccated sodium sulphite 3 ounces; water 8 ounces, hydrochinon 150 grains. Dissolve separately, and mix to form stock solution A. Water 16 ounces, potassium carbonate 2 ounces, potassium bromide 60 grains, forming stock solution B. For use, mix 3 parts of A with 2 parts of B.

bottom of the tray. The tray is rocked in a cradle adapted to give uniform amplitude of motion. Four ounces of developer is a suitable quantity, and it should not be used for more than two or three plates.

The amount of exposure is of the first importance, and is regulated by varying the opening of the iris diaphragm, or by moving the lantern on its supporting bar. As is well known, contrast in a photograph increases rapidly with exposure up to a certain point; and this is the point sought. But as this point lies much below what is known to photographers as "normal exposure," the negative is rather thin, and must be intensified. Intensification further accents contrast when printed, but gives a dense negative very slow in printing. This is of no consequence, however, and the prints show surprising contrast. Considerably less than one per cent. change in illumination on the plate is clearly shown in the prints. Suitable intensification is obtained by bleaching with dilute solution of mercuric chloride and potassium bromide, and blackening with very dilute ammonia in the usual way.

The best amount of light and time of development are determined as follows: Starting with the diaphragm fully open, and with the lamp a few inches from the plate, a series of eighteen spots on a plate are successively exposed, each spot with 10 per cent. less light than its predecessor. This is affected by moving the lantern on its supporting bar away from the plate a predetermined distance for each spot. These distances are fixed by a series of holes in the bar in which a peg is successively placed, to which the lantern is moved. Thus the light is reduced from six to one, in seventeen steps of equal relative amount. Several plates, say five, are exposed in this manner, and, after at least twenty minutes, are developed different lengths of time, say four, five, six, seven and eight minutes, at or near a standard temperature. All are intensified, and printed at a uniform distance from a standard light. Several prints from each plate are made with different times of exposure. Starting with just enough exposure to blacken moderately the weakest spot, the time is doubled for each successive print until the period reaches, say, two hundredfold the time of printing an ordinary pictorial negative.

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Fig. 2.

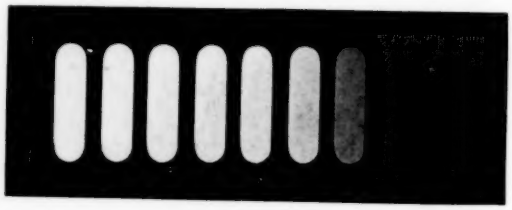
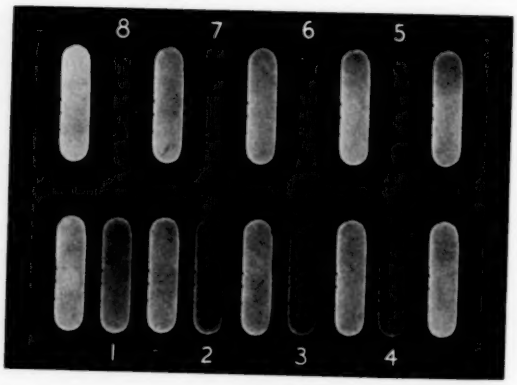


Fig. 3.

Fig. 4.



BRUSH.

PLATE I.

Photographic Photometry.

On comparing all the prints, those from one negative will show the greatest contrast between contiguous spots, thus indicating that the development of this one was correctly timed, or nearly so. The prints from this negative will show also the two adjoining positions occupied by the lantern when the greatest contrast is obtained. One of these positions, or a point between them, is chosen as the standard for subsequent use. It is well to confirm both light and development by further experiments with fewer plates and prints.

If all of the lantern positions are found to give too much or too little light, the diaphragm opening or lamp must be changed, in the direction indicated, and another set of experiments made.

Fig. 2 is a print made from one row of spots on a negative exposed in 10 per cent. steps as described. The left hand spot is, in this case, correctly exposed. The second spot has received 90 per cent. as much exposure; the third spot 81 per cent., the fourth spot nearly 73 per cent. and so on to the ninth spot which has had 43 per cent. as much exposure as the first. Thus each of the last eight spots was exposed to 10 per cent. less light than the spot on its left.

Fig. 3 is another print from the same row of spots, printed about *four hundred times* as long as Fig. 2. The first spot has just begun to darken, while the second is virtually obliterated, and the others entirely so. Here is nearly the whole range from white to black brought about by 10 per cent. decrease in light. The negative of Figs. 2 and 3 illustrates forcibly the growth of contrast with increase of exposure below the "certain point" above alluded to.

In Fig. 4 all the odd-numbered or "comparison spots" in both rows received the same exposure; while the numbered intermediate spots received progressively from 1 to 8 per cent. less exposure. The numbers may be read as percentages of decrease of light represented, in terms of the standard spots.

It is highly important that the temperature of the sensitive plate be maintained nearly constant until all the spots are exposed. Fig. 5 illustrates this. The plate-holder was provided with a metal slide, so as to permit the plate to acquire the laboratory temperature rapidly. When it was certain that this was accomplished

(one hour) the holder was put in place and all comparison spots in both rows were exposed in rapid succession. Then the slide was closed, the holder removed from its place, so as to allow free circulation of air around it, and the room was cooled and kept cooled as nearly as possible 2° C. The measuring thermometer was always hung with its bulb close to the metal slide covering the plate. After half an hour the intermediate spots in the upper row were exposed. Then the cooling process was repeated at 3° below the initial temperature, and the intermediate spots of the lower row were exposed. Contrast due to 2° fall in temperature is well shown, and the difference between 2° and 3° is clearly indicated. In practice, the temperature of the plate is not permitted to change more than 0.1°.

It is probable that the humidity of the plate affects its sensitiveness. This has not been investigated; but to guard against error which might arise from change in humidity between exposure of the comparison spots and the last of the intermediates, which is sometimes an hour, it is customary to keep a supply of plates, in their holders with slides open, in a ventilated dark-box through which the air of the room circulates.

Fig. 6 illustrates both the sensitiveness of this method of photometry and the necessity of careful attention to the maintenance of constant light. A carbon filament lamp was used. The E.M.F. at the lamp terminals was 66.45 volts. After exposing the comparison spots in both rows, a predetermined short length of bare copper wire was cut into the lamp circuit, sufficient to reduce the E.M.F. at the lamp 0.1 volt, and the intermediate spots in the upper row were exposed. Then the E.M.F. was reduced 0.2 volt, and the intermediate spots of the lower row were exposed. The changes in E.M.F. were carefully verified with a very sensitive voltmeter.

Contrast is obvious in both rows, and the difference in effect between a voltage drop of 0.1 volt and 0.2 volt is clearly shown.

A tungsten filament lamp has been much used. This, of course, largely eliminates error due to slight changes in voltage because of its opposite temperature-resistance coefficient. But the tungsten lamp must be used with caution, because expansion of the several lengths of filament, when heated, leaves them free to change shape

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Fig. 5.

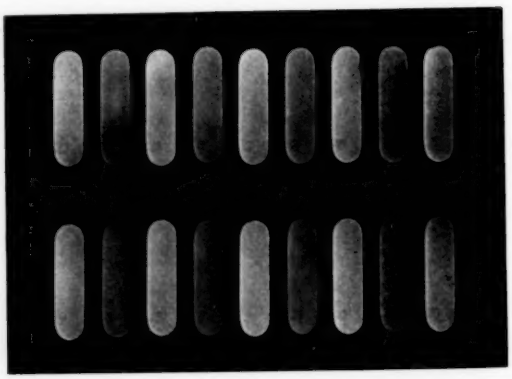


Fig. 6.

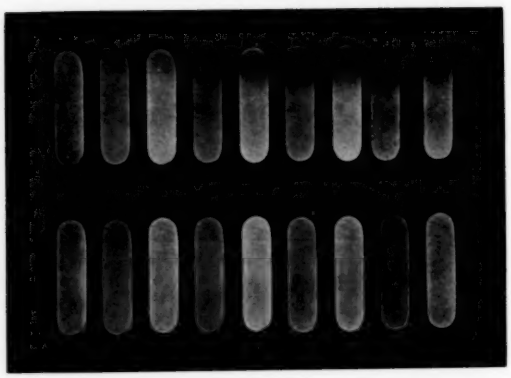
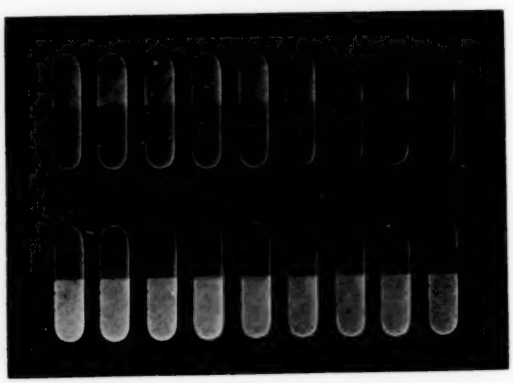


Fig. 7.



BRUSH.

PLATE II.

Photographic Photometry.

at the slightest jar, and thus change position of their own shadows as well as that of the supporting column of the upper spider. A tungsten lamp may be used with safety only after careful testing in different rotational positions, with very gentle tapping after each lighting, has discovered a position giving constant results.

The method of photographic photometry for small light changes as outlined above seems simple and trustworthy. And it *is*, after certain sources of error, not yet touched upon, are understood and eliminated. (They have been eliminated from the experiments thus far described.)

These sources of error lie in the behavior of the latent images before development, and are indicated in what follows.

In the early stages of this work it was observed that intermediate spots exposed with exactly the same light used for the previously exposed comparison spots, always showed some contrast with the latter—they were not quite so bright. This was traced to what, for lack of a better term, is called unequal cross-halation.

It was realized, from the start, that during the exposure of any spot some light would be reflected to neighboring parts of the film from the back surface of the glass, from the back of the plate holder and from the surface of the screen in front of the plate through which the exposure is made, though the latter two were painted dead black. And it was assumed that the effect of this "halation" would be the same on the blank spaces between the comparison spots as it would be on the comparison spots themselves when the intermediates were exposed. In other words, it was assumed that (say) 2 units of light on the fesh film plus 100 units after an interval of time, would produce the same photographic effect as 100 units plus 2 units after an interval. This, however, is very far from being the case. Hence the contrast referred to.

Fig. 7 illustrates the phenomenon under consideration. The 9 spots in the lower row were exposed at intervals of 2 minutes, beginning at the left. The center of the plate was then covered with a strip of cardboard placed so as to shield the upper half of the lower row of spots already exposed, and the lower half of the space on which the upper row of spots were subsequently exposed. Forty seconds after exposure of the last spot in the lower row, the

partially covered plate was exposed 2 seconds to the light from a small orange-red developing window at a distance of 5 feet. After 8 minutes the upper 9 spots were exposed in rapid succession, and 30 minutes later the plate was developed. This plate is printed rather dark, to show the great effect of the feeble extra exposure in the lower row. Observe that the neighboring halves of the two rows of spots have received normal exposure only; while the lower half of the lower row has received feeble extra exposure *after* normal exposure, and the upper half of the upper row has received the same extra exposure *before* normal exposure. The difference in effect is very striking.

By the evidence of many plates, exposed in various ways, the writer is convinced that this difference in effect is not due to abnormal sensitiveness in the fully exposed spots, but to initial insensitiveness of the unexposed places. It seems that a considerable amount of light energy is absorbed in *starting* photographic action, most of which leaves no permanent record. If this is instantly supplemented by more light, as in ordinary exposure, the effective value of each increment of exposure increases until full activity is reached. If, however, the supply of light is cut off as soon as permanent action commences, the starting action quickly relapses, and must be done all over again when subsequent exposure is made. Thus, with the plate in Fig. 7, the starting effect of the weak light in the upper row relapsed almost wholly in the 8 minutes which passed before the normal exposure of the spots took place. That it wholly relapsed within the following 30 minutes, preceding development, is shown by the fact that there is no trace of a fog line on the plate.

The lower row of spots shows that no relapse of normal effect nor change in sensitiveness of the spots occurred during the 16 minutes occupied in exposure. Another plate, just like that of Fig. 7 except that 22 hours intervened between normal exposure of spots in the lower row and secondary exposure to weak light, shows that, during the 22 hours, the starting action in the upper row wholly relapsed; that the normal action in the lower row relapsed very considerably in amount, and the spots lost very greatly in sensitiveness to the weak light. In other words, in the matter

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Fig. 8.

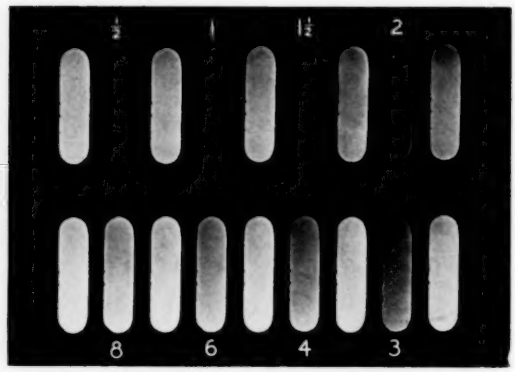
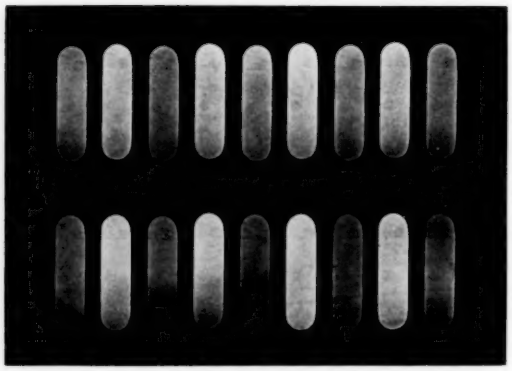


Fig. 9.



BRUSH.

PLATE III.

Photographic Photometry.

of sensitiveness, the spots had gone a long way back toward the normal sluggishness of the unexposed plate.

Similar plates, made with increasing amounts of secondary exposure, indicate that photographic activity increases rapidly after a start is effected, and soon becomes uniform. Thus, spots exposed to 20 units of light followed by 80 units after an interval of several minutes, cannot be distinguished from those exposed to 80 and 20 units with the same interval; nor does either kind differ from those exposed to 100 units all at once.

Some of these phenomena undoubtedly are involved in the early practice of light-fogging a slow plate to increase its sensitiveness.

In the method of photometry first outlined, "unequal cross-halation" is sufficiently suppressed by covering the back of the plate-holder with black velvet pressed tightly against the plate; covering the screen in front of the plate also with black velvet; moderately separating the exposed spots on the plate, and reducing their width so as to reduce the amount of light available for halation. Under these circumstances no "unequal cross-halation" is observable. That it still exists, however, there can be no doubt. This is beneficial, rather than otherwise, in aiding and making visible legitimate contrasts so small as to be uncertain or invisible without it.

The necessity of preventing any leakage of light around the plate-holder, to the plate, between exposure of the comparison spots and the intermediate test spots, is obvious.

Another phenomenon, very interesting to the writer, and quite unexpected, remains to be considered.

It was noticed that plates developed soon after exposure of the last test spot often showed spurious contrast, sometimes very large. This was traced to a continuance, or growth of photographic action *after exposure*, lasting many minutes. Development arrests the growth. Fig. 8 illustrates both propositions.

The comparison spots (Fig. 8) in both rows were exposed in quick succession. After 13 minutes, exposure of the intermediate spots was begun, starting at the left end of the lower row of intermediates, and ending at the left end of the upper row. The time intervals were 2, 2, 1, 1, ½, ½ and ½ minutes. Thirty seconds

after the last exposure the plate was put in the developer, and developed the usual six minutes. The figures on the print represent the time in minutes between exposure of each intermediate spot and beginning of development. The contrast between the spot marked 8 and its neighboring comparison spots indicates the growth of action during the next 13 minutes. It is probable that the growth reaches its maximum in 15 to 20 minutes. In practice, the plates are not developed until at least 30 minutes after the last exposure.

Fig. 9 shows the relapse which probably begins within an hour or two after exposure. The comparison spots in both rows were exposed in the usual manner. After 26 hours, taking care to preserve the original conditions, the intermediate spots in the upper row were exposed. Four hours later the intermediate spots in the lower row were exposed, and in 30 minutes the plate was developed. Contrast in the lower row thus shows the relapse in 30 hours; in the upper row the relapse in the last 26 hours (30 hours minus first 4 hours relapse), and contrast between the rows of intermediate spots shows the relapse, about half of the whole, in the first 4 hours.

In the writer's opinion, this phenomenon of rapid growth in photographic action after exposure, and subsequent rather rapid decay, lends support to the molecular-strain theory of the latent image.

If, in photometric work, the interval between exposure of the comparison spots and the last intermediate spot is more than an hour, it is well to postpone development until the following day.

In conclusion, it may be said that photographic photometry may safely and usefully be employed for detecting and estimating moderate and small changes in light down to amounts considerably less than one per cent. of the whole.

With the plates used by the writer, photographic action is slow in starting, involving considerable light energy which leaves no permanent record. If exposure is stopped at this stage, the starting action relapses almost wholly within a few minutes and is lost to further exposure. Once started, however, action increases rapidly to full activity and then remains substantially constant during further exposure. Relapse of sensitiveness follows exposure, and is very marked at the end of a few hours.

Photographic action, when fully excited, *continues many minutes after exposure*, gradually dying out. With the adopted period of exposure, this after-action amounts to something like eight or ten per cent. of the whole. Within an hour or two after action has ceased, relapse sets in and amounts to about four per cent. in the first thirty hours, fully half of which occurs within the first four hours.

CLEVELAND, OHIO,
December, 1909.

MAGNETIC PROPERTIES OF HEUSLER ALLOYS.

BY EDWARD BEATTIE STEPHENSON.

INTRODUCTION.

THE Heusler alloys are composed of copper, manganese and aluminum and are remarkable for the fact that in certain proportions, they are ferromagnetic, although the component metals are generally considered to be non-magnetic. Since their discovery by Dr. Heusler[1] they have been investigated for most of the phenomena usually accompanying ferromagnetism. The relation of intensity of magnetization to composition,[2] the value of the induction and permeability,[3] the effect of quenching,[4] the critical temperature,[5] magnetostriction,[6] thermoelectric power and Young's Modulus,[7] variation of permeability with composition and quenching temperature,[8] reversibility of the heat effect,[9] susceptibilities of certain salts of copper, manganese and aluminum,[10] the Hall effect,[11] crystalline structure,[12] and the effect of temperature on hysteresis,[13] have all been studied more or less in detail. In general, alloys of the approximate composition of 12 per cent. aluminum, 25 per cent. manganese and 63 per cent. copper were found to be the most magnetic and to have about the same magnetic properties as cast iron, although they varied largely with the previous heat treatment.

[1]Heusler, Verh. d. deut. Phys. Ges., vol. 5, p. 219, 1903.
[2]Heusler, Stark, Haupt, Verh. d. deut. Phys. Ges., vol. 5, p. 224, 1903.
[3]Flemming and Hadfield, Roy. Soc. London Proc., ser. A, vol. 76, p. 271, 1905.
[4]Ross, Roy. Soc. Edinburgh Proc., vol. 27, p. 88, 1906–07.
[5]Gray, Roy. Soc. London Proc., ser. A, vol. 77, p. 256, 1906.
[6]Austin, Ber. d. deut. Phys. Ges., vol. 6, p. 211, 1904.
[7]Guthe and Austin, U. S. Bur. Stds., Bul. 2, p. 297, 1906.
[8]McLennan, PHYS. REV., vol. 24, p. 449, 1907.
[9]Hill, PHYS. REV., vol. 21, p. 335, 1905.
[10]McLennan and Wright, PHYS. REV., vol. 24, p. 276, 1907.
[11]Zahn and Schmidt, Verh. d. deut. Phys. Ges., vol. 9, p. 98, 1907.
[12]Knowlton, PHYS. REV., vol. 30, p. 123, 1910.
[13]Knowlton and Clifford, PHYS. REV., vol. 30, p. 125, 1910.

Preparation.

The alloys used in this investigation were cast by the author in the university foundry in the form of rods 1 cm. in diameter and 30 cm. long. Two rods were cast from each melt, which had been made up in a definite proportion, and then the exact percentage composition was determined by a careful analysis and is shown in Table I. There is a higher per cent. of aluminum that was intended, probably because the aluminum was put in last, melted quickly, and the melt was poured before it had time to oxidize much, while the copper and manganese had to be heated for a much longer time at a higher temperature. The table shows a series of alloys with increasing amounts of manganese. One specimen has iron in it and another a little lead to test the effects of these.

Table I.

No.	Per Cent. Cu.	Per Cent. Mn.	Per Cent. Al.	Per Cent. Pb.	Per Cent. Fe.	Per Cent. IM.[1]
1	74.71	13.41	11.88			
2	69.78	12.48	17.74			
3	61.68	22.57	13.63	1.51		.61
4	54.83	28.11	17.06			
5	66.44	14.47	16.57		2.52	
6	62.43	23.39	14.18			

The specimens were too hard and brittle to be turned in a lathe, so they were ground on an emery wheel to the form of prolate ellipsoids and tested by the magnetometric method, which seemed to be the one best adapted for all parts of the investigation. The magnetizing coil was wound on a copper water-jacket about one meter long, which served to maintain a uniform temperature for ordinary testing and to protect the magnetizing coil when the specimen was heated to high temperatures. For the high temperature work an electric furnace was made by winding nickel wire, bifilar, on a porcelain tube, pasting it over with magnesite and surrounding it with a fireclay tube. The inner tube was just large enough to admit the test piece and the whole furnace fitted inside the water-jacket of the magnetizing coil. The bifilar winding eliminated any field due to the coil of the furnace and the nickel soon lost its magnetic

[1]IM is matter insoluble in HCl or HNO_3.

effect as the coil was heated. The magnetic length of the specimen was assumed to be two thirds the actual length and the values of H were corrected for the demagnetizing effects of the ends.

MAGNETIC TESTS.

The original hysteresis curves for the specimens, which had been allowed to cool slowly in large sand molds and had no further heat

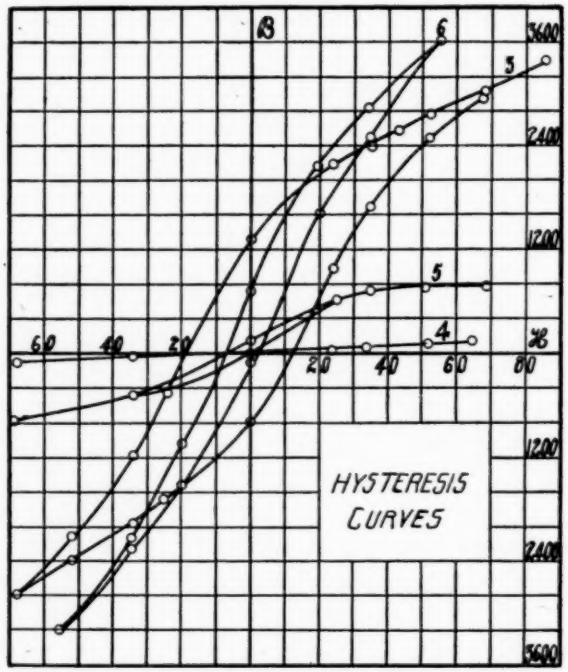


Fig. 1.

treatment, are shown in Fig. 1. Specimens 1 and 2 were so nearly non-magnetic as to be beyond the sensitiveness of the apparatus, although they were paramagnetic. Number 6 was the best in the series as it shows the largest value for the induction, B, with the least hysteresis. Table I. shows that it comes the nearest having the manganese and aluminum in the proportion of their atomic weights. Number 3, with the lead in it, shows about three fourths the value for B that number 6 does and has much greater hysteresis. The iron in number 5 does not seem to have had much effect.

Number 4 with the large per cent. of manganese is very poor magnetically.

From a consideration of atomic susceptibilities,[1] the susceptibilities of certain salts of copper and manganese,[2] and the molecular magnetism of cobalt, iron and manganese, which has been shown to increase in that order,[3] it would seem that manganese may really be a ferromagnetic substance if it can be brought into the right molecular condition, and possibly the alloying with copper and

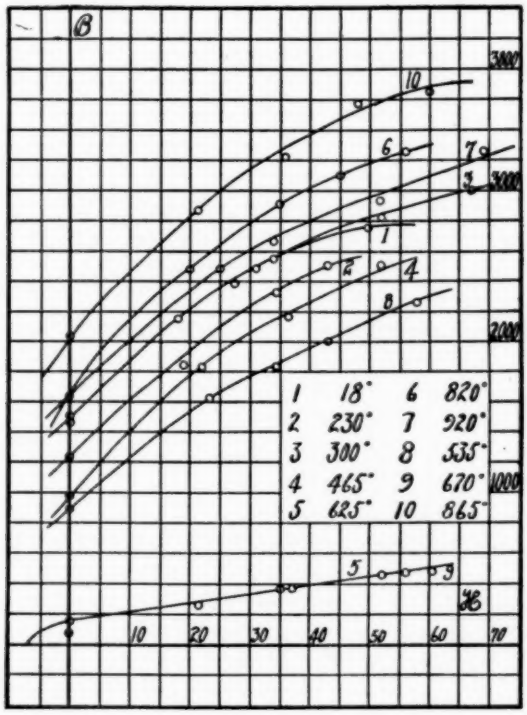


Fig. 2.

aluminum does this. Also, in a way analogous to iron,[4] the alloy or the magnetic constituent of it may have allotropic forms which would have different properties at different temperatures.

[1]Myer, Ann. d. Phys., vol. 68, p. 325, 1899.
[2]Liebknecht and Wills, Ann. d. Phys., vol. 1, p. 178, 1900.
[3]Yager and Myer, Wiener Ber., vol. 106, pp. 504, 623, 1897; vol. 107, p. 5, 1898.
[4]Mellor. Crystallization of Iron and Steel, pp. 12-15.

In order to test the effect of low temperatures, the magnetic specimens 3 and 6 were immersed in liquid air for some time, quickly placed in the magnetometer, and readings taken as the specimen warmed. The low temperatures increased the value of B and reduced the hysteresis, but the improvement was not permanent. The same specimens were then tested at higher temperatures by means of the electric furnace which fitted inside the magnetizing coil. The value of the intensity of magnetization, I, decreased rapidly above 150° C. and entirely disappeared at 300°, but it

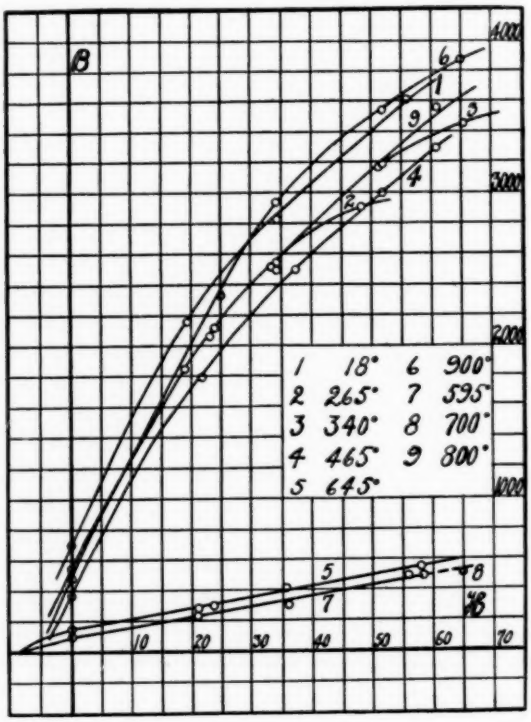


Fig. 3.

regained almost its former value on cooling slowly to room temperature. This effect was reversible up to 350° for slow cooling, but on heating to 500° and cooling more rapidly the value of I fell to about 20 per cent. its former value. This agrees with the results of Hill.

Quenching Tests.

As a further test at high temperatures, quenching in liquid air, in water, and simply cooling in air were tried, on the theory commonly applied to the tempering of iron, that rapid cooling from a given temperature keeps the substance in the crystalline condition that it was in at the temperature from which it was quenched. Cooling in air was found to have the same effect as quenching in water from a somewhat higher temperature, and because this method did not crack the specimens as water quenching did, it was used in the study of specimens 3 and 6. The general method was to heat the specimen in an electric furnace to the given temperature, keep it constant for at least fifteen minutes, and then cool it in air. The

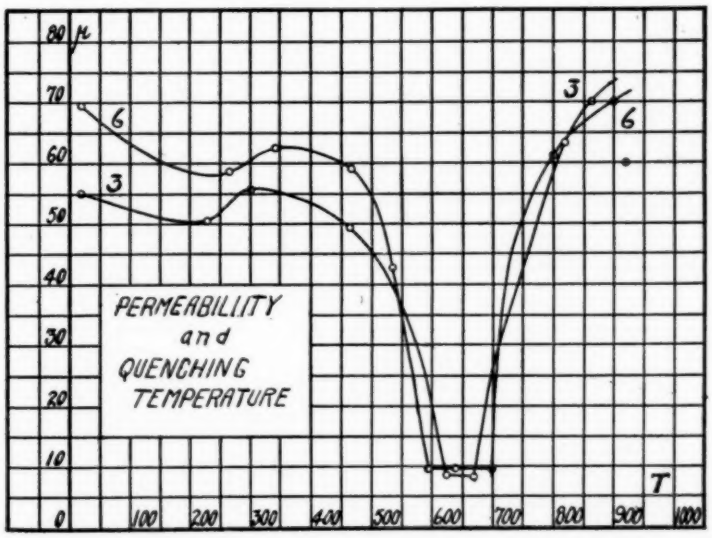


Fig. 4.

results of these tests are shown in Fig. 2 and Fig. 3. Only the upper fourths of the hysteresis curves are shown and the numbers are in the order of quenching. There are two very evident groups of curves in each figure—those cooled from between 600° to 700° being distinctly separated from those above and below that region. To show this relation in another way, the permeability for $H=50$ is plotted against temperatures in Fig. 4. There is a slight drop in the curve around 250° and a very pronounced minimum between 600° and 700°.

Cooling Curves.

Evidently the alloy quenched from between 600° and 700° is different magnetically, and if this is due to the occurrence of an allotropic form, the transition may be accompanied by

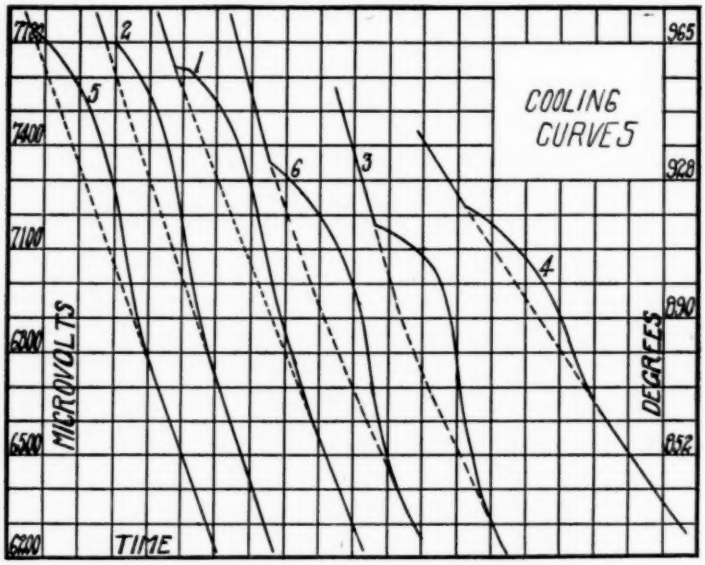


Fig. 5.

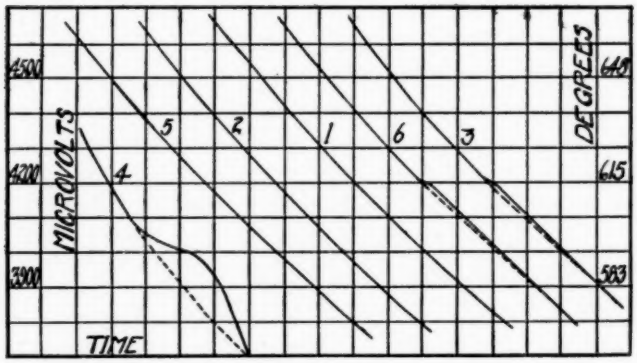


Fig. 6.

some heat change. To test this the data for a series of cooling curves were taken. The specimen was heated considerably above its melting point, then allowed to cool under uniform conditions and temperature readings taken every twenty seconds with a ther-

mocouple. The upper parts of the curves are shown in Fig. 5 and the lower parts in Fig. 6.

It will be seen in Fig. 5 that the curves near the freezing point are those characteristic of solid solutions. Fig. 6 shows that the practically non-magnetic specimens 1 and 2 have perfectly smooth curves, while the magnetic specimens 3 and 6 show a transformation point with a small evolution of heat at 615°, which is about the temperature for which they were non-magnetic when quenched. Curve 4 seems to be abnormal and although the curve shown was undoubtedly obtained, it could not be repeated. The other curves were repeated at least once and the freezing points and transition points found to be quite constant. Evidently there is an allotropic form of these alloys which is non-magnetic.

Crystalline Structure.

The crystalline structure was also studied by taking a large number of photomicrographs, but no definite relation between it and the magnetic properties could be determined. A number of representative pictures are shown in Fig. 7 to Fig. 14, Plates I. and II., the condition under which each was taken being given with the picture.

Summary.

In conclusion the results of this investigation may be summarized as follows:

1. The most magnetic specimens had the manganese and aluminum in approximately the proportions of their atomic weights.

2. Heating to various temperatures, cooling in air and testing at room temperatures, gave for $H=50$, a permeability-temperature curve with a slight minimum near 250°, a very pronounced minimum between 600° and 700° and a rise to better than the original value at 900°. This series of changes was reversible for at least three times.

3. The largest value of induction was obtained when the specimen was quenched in water from near its melting point.

4. A series of cooling curves gave melting points between 910° and 970°, and curves characteristic of solid solutions.

5. The cooling curves for the two magnetic specimens showed transformation points with evolution of heat, at 615°. Specimens cooled in air from this temperature are non-magnetic at room temperature.

6. It seems evident that there is an allotropic form of the alloy which has a practically non-magnetic molecular arrangement.

7. A series of photomicrographs showed differences in crystalline structure due to thermal treatment, but no certain relation could be established between the magnetic properties and crystalline structure.

The thanks of the author are due to Professor A. P. Carman for aid and facilities and to Dr. Chas. T. Knipp for his constant advice and kindly supervision of the work.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
May 13, 1910.

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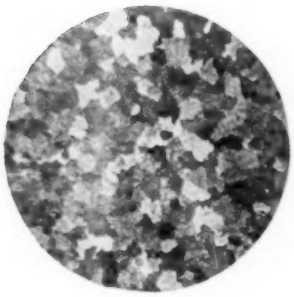

FIG. 7. Specimen 1, twice recast, no further heat treatment. Etched with $FeCl_3$. Paramagnetic. × 50.

FIG. 8. Specimen 2, twice recast, heated for three hours at 575° and quenched in water. Etched with $FeCl_3$. Paramagnetic. Entire section × 10.

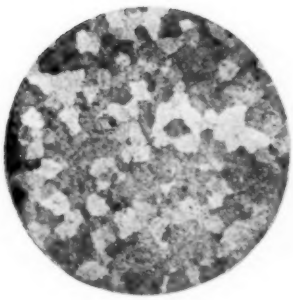

FIG. 9. Specimen 3, twice recast, no further heat treatment. Etched with $FeCl_3$. Strongly magnetic. × 50.

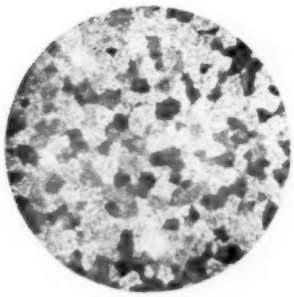

FIG. 10. Specimen 3, twice recast, heated for thirty minutes at 465 degrees and quenched in water. Etched with $FeCl_3$. Slightly magnetic. × 50

STEPHENSON.

PLATE I.

Magnetic Properties of the Heusler Alloys.

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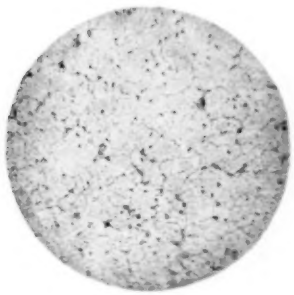

FIG. 11. Original specimen 5 heated for two hours at 620° and quenched in water. Etched with HNO_3. Slightly magnetic. × 50.

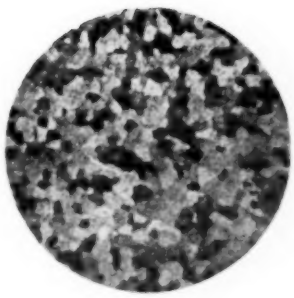

FIG. 12. Specimen 5, twice recast, no further heat treatment. Etched with $FeCl_3$. Slightly magnetic. × 50.

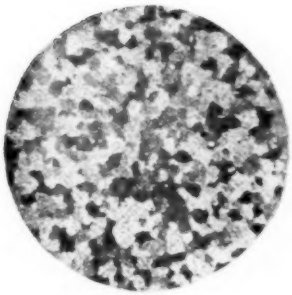

FIG. 13. Specimen 6, twice recast, heated for three hours at 500° and quenched in water. Etched with $FeCl_3$. Strongly magnetic. × 50.

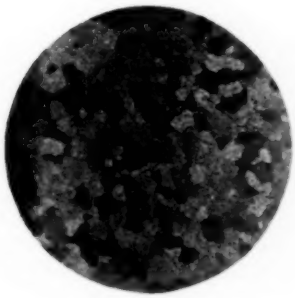

FIG. 14. Specimen 6 as in Fig. 13, heated for thirty minutes at 470° and quenched in water. Etched with $FeCl_3$. Only slightly magnetic. × 50.

STEPHENSON.

PLATE II.

Magnetic Properties of the Heusler Alloys.

TEMPERATURE COEFFICIENTS OF ELECTRICAL RESISTANCE. II.

By Albert Alexander Somerville.

THE work is a continuation of that described in the PHYSICAL REVIEW, Vol. XXX., No. 4, in which data were given on nickel, nichrome, tungsten, and molybdenum. The materials that have been tested since are aluminum, silver, iron, copper, gold, manganin, constantan, "advance," porcelain, quartz, mica, glass, carbon, "excello," "ia ia," "superior," and magnesium. The temperature range is from 0° C. to 1000 or 1100° C. or to the melting point of the specimen. The work divides into four parts:

1. Attaining the desired temperatures.
2. Measuring the temperature at any time.
3. Measuring the resistance of the specimen.
4. Preventing the specimen from oxidizing.

The same electrical resistance furnace has been in use for more than half a year and has been heated to 1000° C. on an average of about five times a week. It consists of co-axial porcelain cylinders 50 cm. in length open at both ends. The inner diameter of the smaller is 3 cm. and the walls are 1 cm. thick; it is wrapped with one fourth inch nichrome ribbon wire and enclosed by the larger cylinder. After a few experiments the outer cylinder was also wrapped with nichrome and the two heating coils connected in parallel. The whole is jacketed with asbestos such as is used for encasing steam pipes. This reaches the first part of the problem, that of attaining a temperature of 1100° C. and in a very convenient manner where a 55- or 110-volt circuit is at command. The rate of temperature change is very easily controlled by placing variable resistance in series with the heating coils.

Callendar's[1] temperature resistance curve for platinum was taken as a standard for making temperature measurements. The plati-

[1] Phil. Trans., vol. 178.

num resistance thermometer has remained practically unchanged during several months' use. Its reading at zero or room temperature was taken each time it was heated, and was frequently read at the temperature of melting copper, that being taken as a fixed point.

The operation of the automatic temperature recorder as described in the first paper in the PHYSICAL REVIEW remained unchanged. This apparatus, when once in order, does best when kept continually working. At present the sensibility is such that the reading across the scale is 400 platinum degrees. A variable resistance in series with the dummy leads of the thermometer enables one to readjust the zero point at any time; knowing the amount of this change one can start to measure temperatures at zero, and when a temperature of 400° is reached, by introducing the proper amount of resistance in series with the dummy leads, bring the pen drawing the curve, back to the other end of the scale and proceed towards 800°; this is repeated to reach beyond 800°. A significant mark or break in the curve is made at any time desired by short circuiting for an instant one arm of the bridge forming part of the recorder. The curve shown in Fig. 1 is a sample of the work done

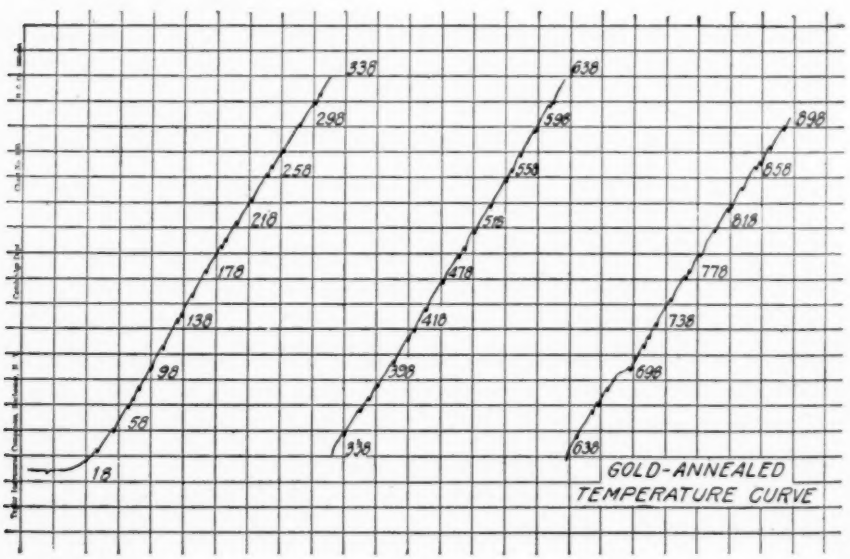

Fig. 1.

when the recorder is in good working condition. The units on the time or X-axis are five minutes each, and the time of a 1000° run is represented as about two hours. The temperature was made to increase at a fairly uniform rate by varying the current through the furnace. Readings indicated on the curve are on the platinum scale.

The transformation from platinum degrees to the Centigrade scale is made by applying the equation

$$T_c = pt + \delta\{(T_c/100)^2 - (T_c/100)\}$$

in which T_c is centigrade degrees, δ, a constant equal 1.5 for standard platinum wire and pt is best defined as being equal to

$$(R_c - R_0)/(R_{100} - R_0) \times 100.$$

Actual measurement of the resistance of a specimen was quite easy, being made on a box bridge, usually reading four decimals, and frequent readings being made on the dummy leads, for which copper has generally been used. Check marks were made on the temperature curve at the same time that a resistance reading was taken and later the temperature at that point was computed.

Probably the most serious inconvenience to working wires at high temperatures is due to oxidation. This trouble has been obviated by equipping a plant for the manufacture of nitrogen and its introduction into the heated region.

The outfit consisted of a nitrogen furnace made by filling a 2-inch iron pipe 2 feet in length with copper shavings, capping both ends, and running small iron pipes from each end so that air could be pumped through. A resistance furnace was built around it so that it could be brought to red heat. A motor-driven pump forced air through this pipe over the hot copper, which took up the oxygen while permitting the nitrogen to pass. In series with it was a wash bottle containing pyrogallol and potassium hydroxide, which served the double purpose of absorbing any oxygen remaining in the gas and indicating the rate at which the gas was flowing. Large bottles to add capacity closed the circuit, so that practically the same gas was pumped continuously through the nitrogen furnace. In parts of the circuit phosphorus pentoxide and calcium chloride were placed to absorb moisture.

When the resistance of a wire is to be measured, it is mounted in a quartz or porcelain tube, one end of which is closed—from the other end extend the lead wires and dummy leads—small quartz tubes are also introduced into the end of the larger tube in order to insulate the wires from each other—also to introduce nitrogen—the end of the larger tube is closed by means of plaster of paris. Then the nitrogen circuit is cut and the tube containing the wire to be measured is made part of the nitrogen circuit—the wire is soon surrounded by an atmosphere of nitrogen and is kept so by allowing a very small stream of the gas to flow continuously. By using ordinary precaution no trouble due to oxidation need be experienced.

The substances tested were those in general most commonly found in a laboratory. The resistance was measured first at the temperature of melting ice—subsequently the temperature was run up to about 1000° C. or 1100° C., or to the melting point of the material. The axes of the curves are *temperature* and *resistance*. For uniformity all resistances are reduced to unity at zero, except those of glass, porcelain, and quartz, in which cases the resistances of a cubic centimeter of the substances at the indicated temperatures are given.

The materials studied may be grouped in various ways. Copper, gold, and silver represent the comparatively pure metals. Not so much can be said of the aluminum and magnesium, but they are much alike. Manganin, constantan, advance, and ia ia form a group of alloys having a small coefficient. Porcelain, quartz, mica, and glass form another class, being non-conductors at ordinary temperatures. Carbon differs from all the others in that the coefficient is always negative. Iron is also in a class by itself. Excello, a good furnace wire put out by Herman Boker and Company, is very much like nichrome, for which data were given in a previous article.[1]

In some cases the wires were drawn through several stone draw plates and the change of resistance due to cold drawing and subsequent annealing was noted. In general, it may be said that cold drawing increases resistance—annealing to a certain extent decreases the resistance by relieving strain, and if carried farther

[1] PHYS. REV., April, 1910.

again causes an increase in resistance due to recrystallization. Successive heatings will cause further crystallization gradually approaching a limit, or if the material is a very hard one the strained condition due to drawing will not be entirely relieved except by continued heating. In rolled ribbon wire the crystals are broken down by the rolling and so lie closer together; there is no strain produced such as in drawing, so that heating at once causes crystallization and a corresponding increase in resistance. The same wire when drawn has a higher resistance, and when first heated this resistance decreases.

Copper.

The temperature coefficient of copper is usually given as about .0040 or .0041. One recent experimenter, who has worked on a

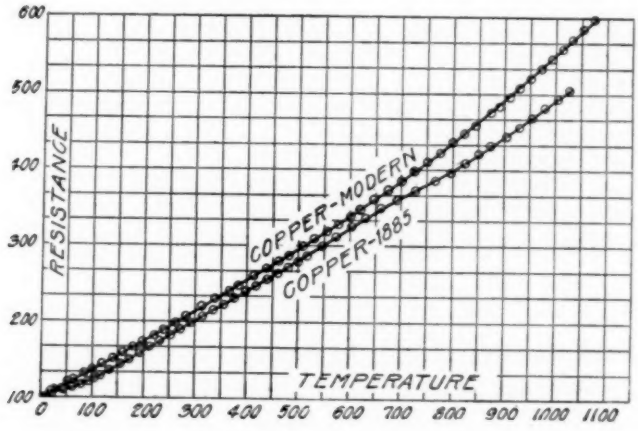

Fig. 2.

half dozen materials, at last report gives it as .0043 at 100° C. and less at 400°, which is contrary to the results obtained by any one else. The author gets various results, depending on whether the wire is hard-drawn or how well it is annealed. If the wire is hard-drawn the first heating will probably cause a decrease in resistance, so that the apparent coefficient is small, possibly only .0037. If the heating causes crystallization, the resistance increases unduly and the coefficient is larger, possibly .0042. Measuring again at zero, however, the coefficient is found to be smaller than that usually given. This refers to modern electrolytic copper, nearly pure.

Copper made twenty-five years ago, before the electrolytic process came to be used, is not nearly so pure, has a smaller coefficient, as is usual with alloys, and also an irregularity in the curve at such a temperature as might indicate the presence of iron.

GOLD.

The effect of cold drawing and annealing is more noticeable in gold than any other substance studied. On this account two curves are shown, giving an idea how great a variation might be

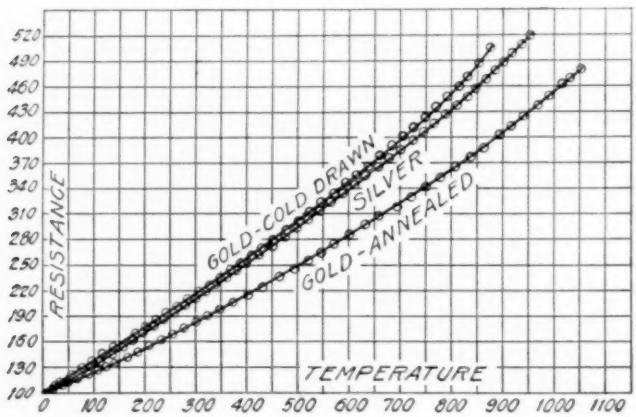

Fig. 3.

obtained in the coefficient. At first it was thought that a trace of silver was causing this small coefficient, but as chemical analysis did not indicate any impurities in subsequent specimens examined, the effects seem to be due entirely to mechanical working and annealing. Lead wires of platinum were used, as the melting point of copper and gold is lower than that of gold, and copper lead wires would melt at a point where they were fused to the gold at a lower temperature than the gold itself. The melting point of gold was used as a check on the temperature as indicated by the thermometer.

SILVER.

Both the gold and silver were obtained from the Philadelphia Mint, the latter in the form of sheets. It was cut into strips, drawn into wires and then annealed. The change in resistance

was not so great as in gold. Silver lead wires were used instead of copper for the same reason as above, that is, that copper lead wires melt off first at the point where fused to the silver. The similarity between the copper, gold and silver curves is to be noted, and also that the electrical resistance is nearly proportional to the absolute temperature.

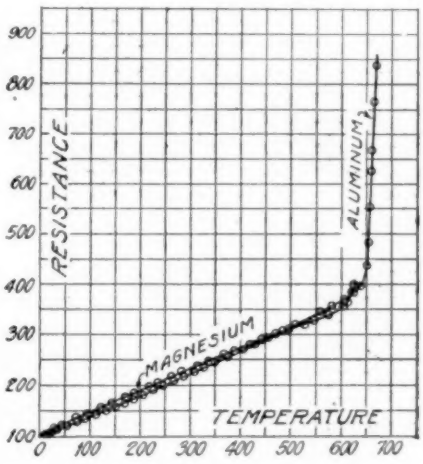

Fig. 4.

Aluminum.

Commercial aluminum of a good grade was tested. Lead wires of the same material were used on account of the difficulty of fusing a connection on to aluminum. The abrupt bend in the curve marks the point where the wire begins to soften, and the last point shows where the wire fell into pieces. The melting point is fairly definite, though not so much so as in the case of some of the other metals. The bend in the curve can be repeated several times with the same specimen provided the temperature is not carried so high that globules begin to form on the lower side of the wire and so permanently alter its shape and cross-section. Magnesium acts in almost exactly the same way as aluminum.

Iron.

A single specimen of iron wire was examined. It was annealed several hours at a temperature of about 1000° C. before any measurements were made on its resistance, and after that annealing scarcely any change was noted in the resistance due to successive heatings. It is to be observed that the coefficient is almost twice as large as that of most metals and increases rather rapidly to a certain point about 785° C., where there is an abrupt change. Over a range of a few degrees at the place where this sudden change occurs the electrical condition is very unstable, and during a change of four

or five degrees it is impossible to measure the resistance, as it suffers such unusual variations in a short time. The recalescence

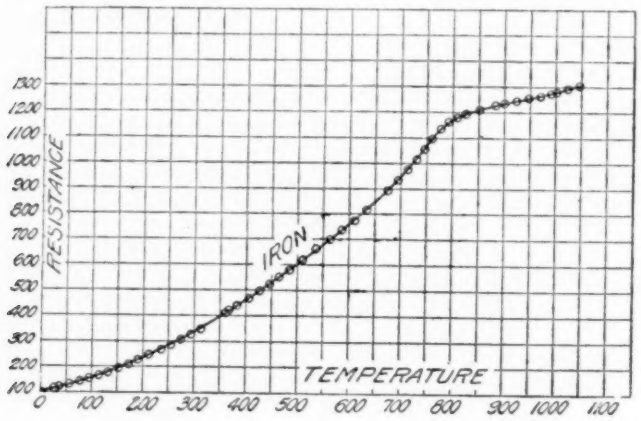

Fig. 5.

point of iron is given by most authorities as 775° C. to 790° C. At this point it ceases to be magnetic. The curve is readily reproducible and there is no hysteresis effect noticeable.

MANGANIN, CONSTANTAN, ADVANCE AND IA IA.

This group of alloys having a small temperature coefficient is interesting when compared. It appears that no mixture yet found will give a zero coefficient, but it may be stated that, as a general

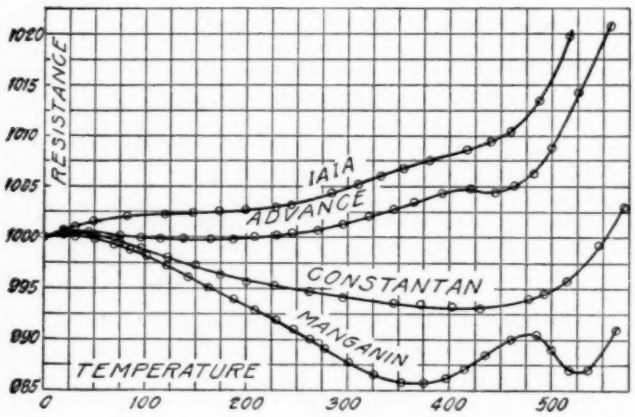

Fig. 6.

rule, the greater the purity of a metal the higher its coefficient, and that mixing metals will not only give a smaller coefficient, but may even cause it to change sign as the temperature varies. This latter fact is taken advantage of, and an alloy made in which the sign of the temperature coefficient of resistance will change at just about room temperature or slightly above this temperature. Thus the slope of the resistance temperature curve at this point is zero. In three of these alloys a maximum occurs at about room temperature, so for a short range of temperature their resistance is nearly constant. The actual increase in resistance from zero to 20° C. is: Manganin, .04 per cent.; constantan, .05 per cent.; advance, .02 per cent.; ia ia, .05 per cent. Constantan then pursues the smoothest course and the total variation up to 600° C. is less than 1 per cent. In the case of manganin the coefficient changes size and sign frequently. At about 850° C. the resistance of advance begins to increase rapidly.

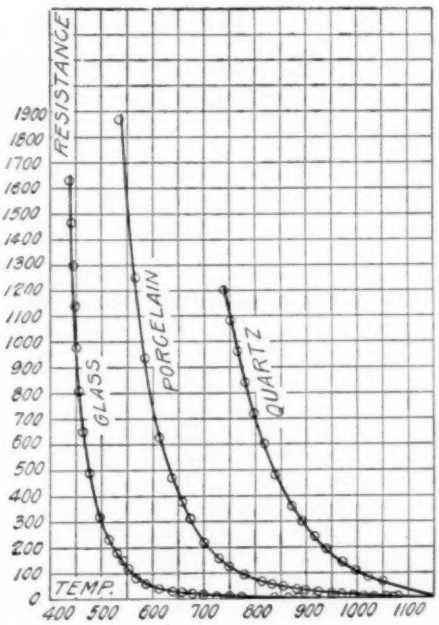

Fig. 7.

Porcelain, Quartz, Mica, and Glass.

These so-called insulators undergo enormous changes in resistance when heated. In each of these substances the resistance begins to decrease at a very definite temperature, and falls rapidly, so that within only a few degrees the resistance may change to one tenth of its former value. However, the resistance is still very high. In the liquid state glass becomes a sufficiently good conductor to carry current large enough to heat it farther and so cause it to fall apart. It is so difficult to secure two

pieces of mica alike that only the general characteristics of the mineral can be represented by a curve. On first heating its resistance varies very much like that of porcelain, but after it has been held at a temperature of about 1100° C. for a few hours, its resistance increases until it is almost a perfect insulator. It has then become white and rather brittle. Presumably all impurities have been driven off; in this condition it makes the best frame known for a resistance thermometer.

In the case of fused quartz, which is made by squirting, if the logarithm of the resistance is plotted against temperature, a straight line results. The resistance of the transparent quartz, which is cast, does not begin to decrease until 1100° is reached and there the curve has a slope very nearly the same as that of the fused quartz at 740°. The Berlin porcelain begins to soften at about 1100° C. and as the resistance thermometer is enclosed in such a tube it cannot be run any higher. The resistance of these materials was measured while suspended in mid-air in the furnace.

CARBON.

Copper terminals were plated onto the ends of a piece of carbon 1 mm. in diameter and 10 or 15 cm. in length. This, like the speci-

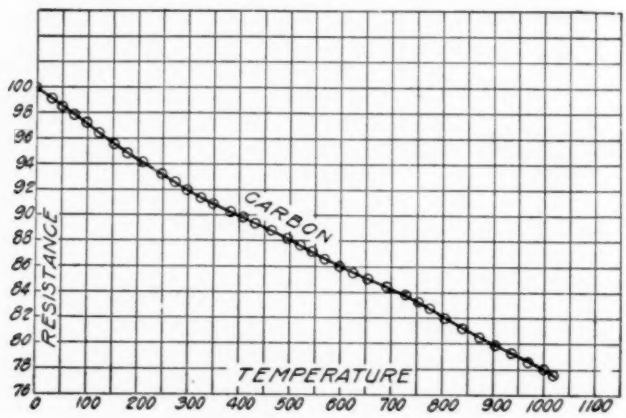

Fig. 8.

mens of wire, was mounted in a porcelain tube, and nitrogen forced in to take the place of air. Two slight peaks are seen in the curve,

these occurring at temperatures of about 450° and 700°. The squirted carbons are supposed to be wholly amorphous or non-crystalline and so have a regular change of resistance. Graphite, another form of carbon, is crystalline, belonging to the hexagonal system. The temperature resistance curve of the graphite taken from a lead pencil shows two sharp peaks occurring at about the same temperatures as these slightly prominent ones in carbon, so it is not improbable that the carbon is, or has become slightly crystalline during the heating.

Excello and Superior.

These are two alloys furnished by Herman Boker and Company. The former, a nickel chromium alloy, does not oxidize in air and does not melt below 1200° C., and is therefore a good furnace

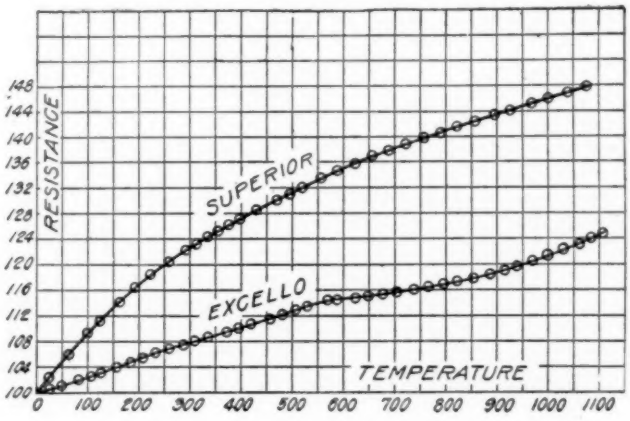

Fig. 9.

wire. It is also pliable, which is an advantage when winding a furnace.

Superior, a nickel-steel alloy, is also a wire that works well. It is evidently thoroughly annealed, as heating to a thousand degrees and cooling to zero again, brings it back almost exactly to its original resistance.

The temperature coefficient, *i. e.*, the slope of the temperature-resistance curves, is plotted against temperature in Figs. 10

to 15 and the values of the temperature at different temperatures are also given in the table at the close of this article.

Some of the apparatus used has been secured by a grant by the Smithsonian Institute. For this and other apparatus used, and for

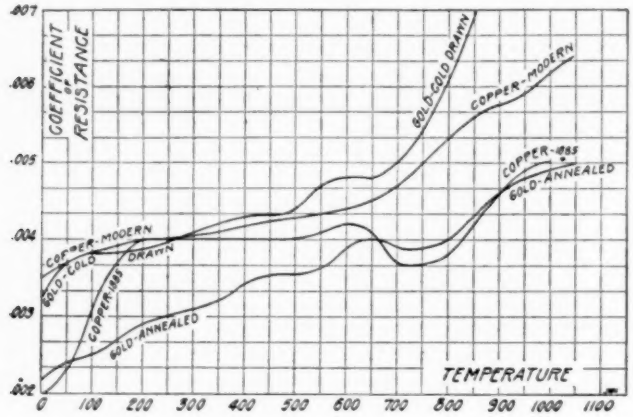

Fig. 10.

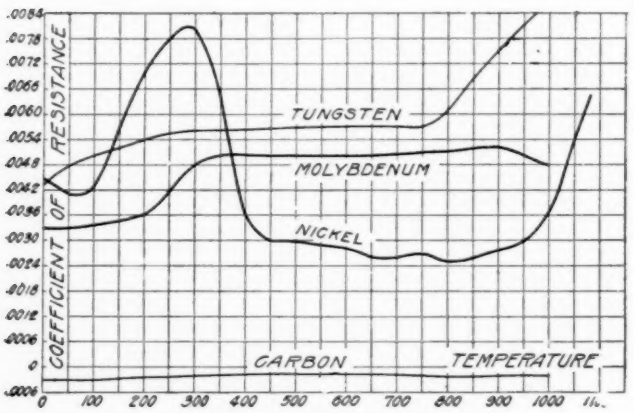

Fig. 11.

the suggestions made by Prof. J. S. Shearer and other members of the Physics Department the auther desires to express his thanks.

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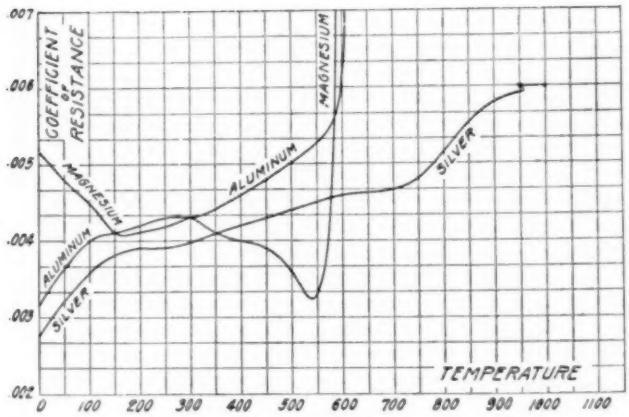
COEFFICIENT OF RESISTANCE
.007
.006
.005
.004
.003
.002
MAGNESIUM
ALUMINUM
SILVER
ALUMINUM
MAGNESIUM
SILVER
TEMPERATURE
0 100 200 300 400 500 600 700 800 900 1000 1100

Fig. 12.

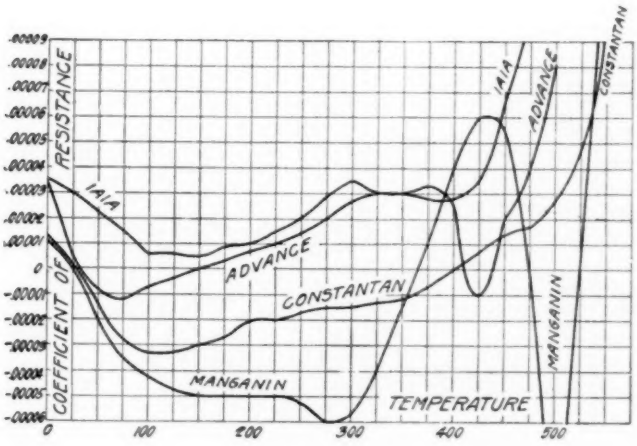
COEFFICIENT OF RESISTANCE
IAIA
ADVANCE
CONSTANTAN
MANGANIN
TEMPERATURE
0 100 200 300 400 500
IAIA
ADVANCE
CONSTANTAN
MANGANIN

Fig. 13.

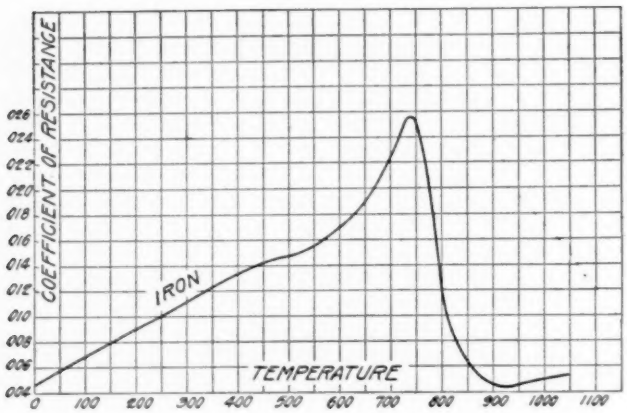
COEFFICIENT OF RESISTANCE
IRON
TEMPERATURE
0 100 200 300 400 500 600 700 800 900 1000 1100

Fig. 14.

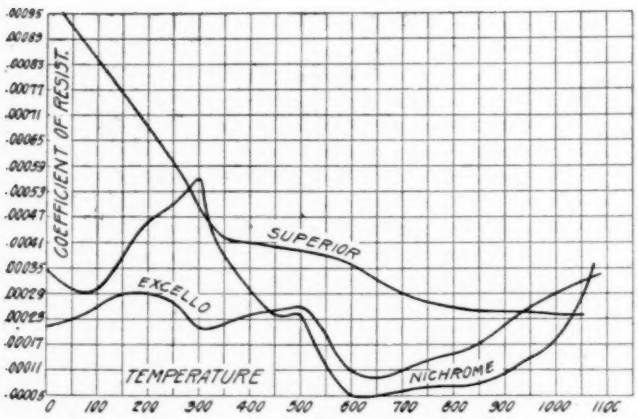
COEFFICIENT OF RESIST.
SUPERIOR
EXCELLO
NICHROME
TEMPERATURE
0 100 200 300 400 500 600 700 800 900 1000 1100

Fig. 15

Temperature Coefficients of Electrical Resistance.

Temperature, Centigrade.	Advance.	Ia/Ia.	Manganin.	Constantan.
12.5	.000020	.000033	.000006	.000008
25	.000005	.000030	.000000	.000002
50	−.000008	.000022	−.000020	−.000014
75	−.000012	.000015	−.000035	−.000028
100	−.000007	.000006	−.000042	−.000033
125	−.000004	.000006	−.000047	−.000033
150	.000000	.000005	−.000050	−.000030
175	.000003	.000009	−.000050	−.000027
200	.000007	.000010	−.000050	−.000020
225	.000010	.000015	−.000050	−.000020
250	.000014	.000020	−.000052	−.000017
275	.000020	.000029	−.000060	−.000015
300	.000027	.000035	−.000057	−.000015
325	.000030	.000030	−.000040	−.000013
350	.000030	.000030	−.000015	−.000012
375	.000033	.000028	.000010	−.000007
400	.000025	.000028	.000040	.000000
425	−.000010	.000035	.000060	.000007
450	.000020	.000067	.000055	.000014
475	.000040	.000110	.000000	.000017
500	.000080	.000200	−.000110	.000027
525	.000105		.000000	.000047
550	.000105		.000150	.000100

Temperature, Centigrade.	Gold, Cold Drawn.	Gold, Annealed.	Silver.	Iron.	Tungsten.	Molybdenum.	Nickel.	Nichrome.
25	.0035	.0023	.0030	.0052	.0046	.0033	.0043	.00032
50	.0037	.0024	.0032	.0057	.0048	.0033	.0041	.00030
100	.0038	.0025	.0036	.0068	.0050	.0034	.0043	.00030
150	.0038+	.0027	.0038	.0079	.0052	.0035	.0057	.00038
200	.0039	.0029	.0039−	.0090	.0054	.0036	.0070	.00046
250	.0040	.0030	.0039	.0100	.0056	.0042	.0078	.00050
300	.0041	.0031	.0040	.0111	.0056	.0048	.0080	.00056
325							.0080	.00043
350	.0042	.0032	.0041	.0123	.0056	.0050	.0064	.00038
375							.0048	
400	.0043	.0034	.0042	.0133	.0056	.0050	.0036	.00030
425							.0032	
450	.0043	.0035	.0043	.0142	.0056	.0050	.0030	.00024
500	.0044+	.0035	.0044	.0147	.0057	.0050	.0030	.00024
550	.0047	.0036	.0045	.0155	.0057	.0050	.0029	.00012
600	.0048	.0039	.0046	.0170	.0057	.0050	.0028	.00005
650	.0048	.0040	.0046	.0189	.0057	.0050	.0026	.00005
700	.0050	.0039	.0047	.0224	.0057	.0050	.0026	.00006
725				.0248				
750	.0054	.0039	.0048	.0250	.0057	.0050	.0027	.00007
775				.0200				
800	.0061	.0040	.0052	.0120	.0061	.0051	.0025	.00007
825				.0080				
850	.0066	.0043	.0055	.0065	.0068	.0052	.0026	.00008
875				.0052				
900		.0046	.0058−	.0046	.0075	.0052	.0028	.00010
950		.0048	.0059	.0046	.0081	.0050	.0030	.00014
975						.0049		
1000		.0049		.0050	.0089	.0048	.0037	.00018
1050		.0050		.0053	.0093		.0054	.00028
1075							.0062	.00036

Temperature, Centigrade.	Aluminum.	Magnesium.	Copper, Modern.	Copper, 1885.	Carbon.	Superior.	Excello.	Quartz.	Porcelain.	Glass.
25	.0034	.0050	.0036	.0021	−.00030	.00096	.00022			
50	.0037	.0048	.0037	.0023	−.00030	.00092	.00023			
100	.0040	.0045	.0038	.0031	−.00031	.00084	.00026			
150	.0041	.0041	.0039	.0037	−.00028	.00076	.00029			
200	.0042	.0041	.0040	.0040	−.00025	.00068	.00029			
250	.0043	.0042	.0040	.0040	−.00024	.00060	.00027			
300	.0043	.0043	.0041	.0040	−.00023	.00049	.00021			
350	.0044	.0041	.0041	.0040	−.00020	.00042	.00022			
400	.0046	.0040	.0042	.0040	−.00018	.00041	.00024			
450	.0048	.0039	.0042	.0040	−.00018	.00040	.00025			−32.00
475										−32.00
500	.0050	.0036	.0043	.0040	−.00020	.00039	.00026			− 6.00
525		.0032								− 3.60
550	.0053	.0033	.0043	.0041	−.00020	.00038	.00020			− 2.50
575	.0054	.0040					.00015		−16.00	− 1.50
600	.0060	.0100	.0044	.0042	−.00019	.00036	.00011		− 9.80	− .80
625	.0120	.0250							− 6.20	− .45
650			.0045	.0041	−.00018	.00032	.00009		− 4.60	− .30
675									− 3.70	− .20
700			.0047	.0037	−.00018	.00029	.00011		− 2.80	− .17
725									− 2.00	− .14
750			.0050	.0037	−.00020	.00027	.00013	−10.00	− 1.60	− .10
775								− 8.02	− 1.00	− .07
800			.0053	.0038	−.00022	.00026	.00015	− 6.40	− .70	− .06
825								− 5.20	− .50	
850			.0056	.0042	−.00022	.00025	.00017	− 4.20	− .40	
875								− 3.20	− .35	
900			.0057	.0046	−.00020	.00025	.00022	− 2.60	− .30	
925								− 2.20	− .25	
950			.0059	.0049	−.00020	.00025	.00026	− 1.80	− .20	
975								− 1.30	− .16	
1000			.0062	.0050	−.00020	.00024	.00029	− 1.00	− .12	
1025								− .80		
1050			.0064			.00024	.00032	− .65		

DEPARTMENT OF PHYSICS,
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THE DIELECTRIC CONSTANT OF ICE, FOR LONG WAVES, AT TEMPERATURES NEAR THE MELTING POINT.

BY PHILLIPS THOMAS.

THE determination of the dieletric constant of ice at comparatively high temperatures offers great difficulty, for the conductivity, even for the purest ice obtainable, is found to be quite large. Ice ceases to behave as a static insulator at about $-130°$ C. No direct method, so far as the author has been able to discover, has proved available for making this measurement with infinite wave-length at points higher than the above temperature.

Several observers have found values for the constant by more or less indirect methods. Bouty,[1] using a torsion pendulum, charged a condenser, with ice dielectric, for a known time, then discharged into a standard capacity and measured the resulting charge, using an equation of the form

$$Q = E(C + at),$$

where a is a conductivity coefficient, Q the quantity discharged into the condenser, t the time of original charge. He found the constant to be 78, at a temperature of $-23°$ C. Perot,[2] with a Blondlot oscillator, working at about the same temperature, found values from 60 to 71, at wave-lengths of from 680 to 1,360 meters. Thwing[3] and Abegg,[4] employing resonators giving much shorter waves, found values in the neighborhood of 3, at temperatures ranging from $-24°$ to 0° C. Dewar and Fleming[5] used a charge and discharge method, with an electrically driven tuning fork, and found values from 2.83 at $-198°$, to 11.0 at $-131°$ C., but were unable to carry the temperatures higher, as the conductivity became too large.

[1]Bouty, Comptes Rendus, vol. 114, p. 533, 1892.
[2]Perot, ibid., vol. 114, p. 1528, 1892.
[3]Thwing, Zeitschr. Phys. Chem., vol. 14, p. 286, 1894.
[4]Abegg, Wied. Ann., vol. 65, p. 229, 1898.
[5]Dewar and Fleming, Proc. Roy. Soc., vol. 61, pp. 2 and 316, 1897.

Static measurement being found impracticable, a "current" method was first tried. A special electrodynamometer was made, of high sensibility: the voltage around this, in series with the condenser under test, was read on an ordinary Weston alternating voltmeter. But the maximum sensibility attainable was so low that in order to be able to read the deflections of the electrodynamometer with any accuracy—on the low frequencies desirable—it was necessary to employ voltages of from 1,000 to 4,500, the magnetizing current for which pushed the densities in the transformer iron high enough to bring out a prominent fifth harmonic in the voltage curves. Upon taking oscillograms of the actual current through the condenser, it was found that the fifth harmonic was much more strongly emphasized, resulting of course in showing an apparent dielectric constant considerably in excess of the actual values.

The method finally adopted was as follows. The capacity to be measured was connected in series with a variable resistance, and the voltages around the two were read, simultaneously, on two Dolezalek electrometers. The latter, being fitted with suspensions of equal cross-section and length, and with needles of equal weight and area, had approximately the same calibration, and the resistance being adjusted to take nearly the same voltage as the capacity on each frequency, the correction factor of the voltmeter by which the electrometers were calibrated was practically eliminated. Also, the required total E.M.F. being only about 150 volts, and the current of the order of 10^{-5} ampere, the voltage and current curves were similar. These were made almost pure sines by passing the current, before reaching the test circuit, through two transformers operated below the knee of their iron.

The paper needles supplied with the electrometers were found to be so heavy that with suspensions small enough to give the desired deflection, their periods became unmanageably long. Accordingly, needles were made of a light aluminium foil, fitted with staves of 0.02″ wire of the same material. The electrometers then had a period of about one second, and ample sensibility, with a 0.002″ phosphor-bronze suspension of 6 cm. length.

THEORY OF THE METHOD.

Let the electrometer connected across the series resistance be called no. 1, that across the capacity, no. 2, and let their capacities be c_1 and c_2, respectively. Let the capacity under the test be C, let r be the value of the series resistance, R the resistance of the condenser under test, and f the frequency employed. Also let x_1 and x_2 represent $1/2\pi fc_1$ and $1/2\pi fc_2$, respectively: let X equal $1/2\pi fC$, and call the readings of the electrometers E_1 and E_2, respectively.

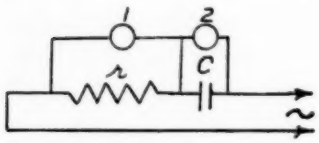

Fig. 1.

Case 1.—Test condenser having inappreciable conductivity With connections as in Fig. 1,

$$\frac{E_1}{E_2} = \frac{Z_r}{Z_C},$$

where Z_r and Z_C are the impedances of the r and C branches,

$$\frac{1}{Z_r} = \left(\frac{1}{r^2} + \frac{1}{x_1^2}\right)^{\frac{1}{2}},$$

and

$$\frac{1}{Z_C} = \frac{1}{X} + \frac{1}{x_2}.$$

Now let

$$d = \frac{E_1}{rE_2},$$

and suppose $1/x_1^2$ negligible compared to $1/r^2$. Then

$$C = \frac{d}{2\pi f} - c_2. \qquad (1)$$

Case 2.—Conductivity of test condenser not negligible. In this case, the test condenser, in addition to its charging current, will take a true ohmic current, in quadrature with the former. With connections as in Fig. 2, as before,

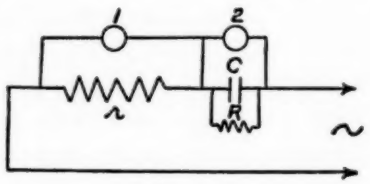

Fig. 2.

$$\frac{E_1}{E_2} = \frac{Z_r}{Z_C},$$

where

$$\frac{1}{Z_r} = \left(\frac{1}{r^2} + \frac{1}{x_1^2}\right)^{\frac{1}{2}}$$

and

$$\frac{1}{Z_C} = \left(\frac{1}{R^2} + \left(\frac{1}{X} + \frac{1}{x_2}\right)^2\right)^{\frac{1}{2}}.$$

If readings be taken on two frequencies f and $\dot{f}$—assuming R, r and C constant for this range of frequency, and replacing E_1/rE_2 by d as before—

$$\frac{1}{R^2} = d^2 - \frac{1}{\left(\frac{1}{X} + \frac{1}{x_2}\right)^2} = \dot{d}^2 - \frac{1}{\left(\frac{1}{\dot{X}} + \frac{1}{x_2}\right)^2}, \qquad (2a)$$

from which

$$\dot{d}^2 - d^2 = \frac{1}{\left(\frac{1}{\dot{X}} - \frac{1}{x_2}\right)^2} - \frac{1}{\left(\frac{1}{X} - \frac{1}{x_2}\right)^2} = (C + c_2)(\dot{w}^2 - w^2),$$

where

$$w = 2\pi f \quad \text{and} \quad \dot{w} = 2\pi\dot{f}.$$

Hence,

$$C = \frac{\dot{d}^2 - d^2}{\dot{w}^2 - w^2} - c_2. \qquad (2)$$

Equation (1) was used only to evaluate capacities with air as dielectric; all values taken on the ice condensers were worked out by equation (2). Thus, both the capacity and the ohmic resistance of the condenser may be found from two sets of readings of the electrometers, at two different frequencies—the resistance being found by substituting the value of C and c_2 in equation (2a). As r, the series resistance, was never more than 3×10^6 ohms, the impedance of the r branch was in no case sensibly different from r, the value of c_1 being about 3×10^{-11} farad, and the frequencies being all between 80 and 20.

Description of Apparatus Used.

Current was obtained at 120 volts from a battery of storage cells of large current capacity. This voltage was applied to the

direct current side of a 2-H.P. rotary converter, giving, from the slip-rings, alternating current constant at 85 volts. The frequency of the latter was varied within the above limits by increasing or decreasing the field current of the rotary. By a combination of transformers and potentiometer resistances, this alternating current was fed to the apparatus at any voltage up to 200.

The electrometers were connected to high-insulation rubber keys, so that both could be thrown on their separate branches of the circuit, simultaneously, or either branch could be short-circuited, allowing instant calibration of the other electrometer. The frequencies were obtained from the reading of a large semi-portable type Weston voltmeter, connected to the terminals of a Holzer-Cabot magneto generator, which was driven by a flexible shaft direct connected to that of the rotary converter.

CALIBRATION OF INSTRUMENTS. ACCURACY OBTAINABLE.

1. *Electrometers.*—These were both calibrated against Weston direct-reading voltmeter no. 250, on alternating current. The voltmeter was checked against the semi-portable standard used in reading frequency and known to be right to 0.02 per cent. Meter no. 250 was found to have a nearly constant correction factor of 0.99, throughout the range used. As the constancy of the electrometer calibrations controls absolutely the accuracy of the method, a check calibration was taken on each of them, after the experimental work had been concluded. This is given below in Table I., intermediate values being omitted for the sake of brevity.

The capacity of each electrometer was determined. That of no. 1 was found to have no appreciable effect on the impedance of its branch of the circuit, as stated above. But c_2 directly modifies the results, and was determined very carefully. The method used and value found are given below, under condenser no. I.

2. *Frequency.*—A set of Koenig forks was employed to calibrate the voltmeter used on the frequency magneto. The current from the rotary was passed through a lamp and sounder in series, the circuit being then tuned to each of the forks in succession, by altering the field current of the rotary. Table II. gives the frequency of each fork, the corresponding voltmeter reading with the circuit

tuned to the fork, and the ratio. Upon re-calibrating after the work, the ratio was found to be the same.

TABLE I.

Electrometer Check Calibrations.

No. 250. Volts.	No. 1, mm. Deflection.		No. 250. Volts.	No. 2, mm. Deflection.	
	May 10.	May 15.		May 10.	May 15.
50	43	44	60	53	53
56	56	58	66	67	67
60	64	65	70	74	75
66	78	79	76	87	88
70	87	88	80	95	96
76	103	104	86	111	111
80	112	113	90	120	120
86	130	132	96	136	136
90	141	142	100	145	146
96	162	161	106	163	163
100	175	175	110	172	174
106	198	198	116	191	192
110	214	214	120	203	203
116	240	239	126	220	220
120	256	256	130	232	231
			136	250	249
			140	262	261

TABLE II.

Magneto Calibration.

Fork Frequency.	Magneto Voltage.	Ratio	Fork Frequency.	Magneto Voltage.	Ratio.
42.50	18.50	2.310	53.32	23.10	2.305
45.70	19.80	2.310	60.00	26.00	2.310
48.00	20.80	2.310	64.00	27.70	2.310
51.20	22.20	2.305	80.00	34.67	2.310

3. *Resistance, r.*—This was taken, in every case, from megohm boxes made by Gambrell, which were calibrated against a Reichsanstalt certified Wolffe bridge. The megohm units were divided in ten parts, each of 100,000 ohms. No one of these was of value above 100,030 ohms, or below 99,960 ohms, at the room temperature. This correction was neglected.

The frequency voltmeter could be read with certainty to only

0.2 per cent. This, then, is the accuracy claimed for the method, although the electrometers would admit of much closer work.

TEST CONDENSERS USED.

These were three in number, and will be designated no. I., no. II. and no. III. The first two were cylindrical, no. I. being 42 cm. long, 10 cm. mean diameter, with 3 mm. dielectric thickness, no. II. 8 cm. long, 3 cm. mean diameter, with 0.8 mm. dielectric thickness. The cylinders were kept coaxial by hard-rubber heads. These two condensers were frozen, in each case, as follows: The condenser was assembled and made water-tight, taken into the refrigerating room and allowed to cool to 0° C. Meanwhile water was freshly distilled and cooled in the same way. The condenser was then filled, through an opening in the top cover, and frozen up from the bottom, being kept melted at the top by passing a slight current around a coil on the outer cylinder. In this way no trouble was met in obtaining the same dimensions when frozen, as in air. Condenser no. I., upon which nearly all measurements were taken, was copper-plated, inside and out, from the same anode, thus securing chemically identical surfaces.

Condenser no. III. was made of two parallel brass plates, separated by 0.4 mm. mica washers. This was frozen in a beaker of distilled water, the condenser standing vertically in the beaker, the whole being surrounded by a heating coil. The plates were 10 cm. square.

The two smaller condensers, no. II. and no. III., were used only to discover any possible effect of the form of condenser or of the dielectric thickness, upon the constant. No. II. was also immersed in liquid air, and its capacity taken at that temperature.

Air Capacity.—Table III. gives the readings on the two electrometers with the circuit made up of condenser no. I., air dielectric, with the calculated value of $C+c_2$. These were then put in series on the voltage, and the fraction of the whole voltage, taken by each, determined, thus giving C/c_2 (see Table IV.). C and c_2 were then calculated, their values being as follows:

$$C = 4.41 \times 10^{-10} \text{ farad.}$$
$$c_2 = 5.28 \times 10^{-11} \text{ farad.}$$

EXPERIMENTAL.

1. *Condenser No. I.*

TABLE III.

Air Capacity, Condenser No. I. + Electrom. No. 2.

Magneto.		r	No. 1.		No. 2.		E_1/E_2	Eqn. (1). $C+c_2$
Volts.	Freq.		Defl.	E	Defl.	E		
20	46.2	3×10^6	50	51.9	208	121.1	.428	
			53	53.7	217	124.2	.433	
			56	55.7	228	128.2	.434	4.95×10^{-10}
			59	57.0	238	131.9	.432	
			62	58.6	248	135.3	.433	
23	53.1	3×10^6	63	59.0	200	118.2	.499	
			66	60.1	208	121.1	.496	
			69	61.3	217	124.2	.493	4.94×10^{-10}
			72	62.4	225	127.1	.491	
			75	63.8	233	130.0	.491	
26	60.0	3×10^6	74	63.4	188	113.9	.557	
			77	64.5	193	115.6	.558	
			80	65.8	200	118.2	.556	4.93×10^{-10}
			83	67.2	207	120.7	.557	
			86	68.2	213	122.8	.556	
29	67.0	3×10^6	86	68.2	176	109.4	.624	
			89	69.4	182	111.7	.623	
			92	70.7	188	113.9	.623	4.94×10^{-10}
			95	72.0	193	115.6	.623	
			98	73.2	198	117.3	.624	

Mean, 4.94

TABLE IV.

Ratio of c_2 to C.

Total Volts.	No. 2.		$E_C/E_2=c_2/C$
	Defl.	E	
124.3	180	111.1	.1190
120.3	170	107.3	.1210
116.0	162	104.5	.1195
112.4	151	100.5	.1185

Mean, .1195

In order to connect with the condensers when frozen, a pair of no. 20 bare copper wires were led to the refrigerating room, being

TABLE V.

Condenser No. 1., Ice Dielectric, Readings.

Magneto.		r	T	No. 1.		No. 2.		E_1/E_2	Mean.
Volts.	Freq.			Defl.	E	Defl.	E		
20	46.2	5×10^4	−2	79	65.4	145	98.4	.666	
				88	69.1	158	103.1	.669	.6705
				95	72.0	169	107.1	.672	
				101	74.5	178	110.3	.675	
30	69.3	5×10^4	−2	121	82.1	116	87.4	.940	
				130	85.0	122	89.9	.945	.9448
				140	88.2	130	93.0	.947	
				152	92.1	142	97.4	.947	
20	46.2	5×10^4	−3	93	71.1	160	103.9	.684	
				103	75.4	185	112.8	.670	.6735
				113	79.2	199	117.9	.672	
				123	82.7	213	122.8	.668	
30	69.3	5×10^4	−3	150	91.5	141	97.0	.945	
				160	94.7	148	99.4	.952	.9523
				170	97.5	156	102.4	.952	
				180	100.4	163	104.8	.960	
20	46.2	5×10^4	−5	92	70.9	181	111.3	.638	
				95	72.1	186	113.0	.638	.6383
				98	73.4	191	114.8	.639	
				101	74.5	196	116.7	.638	
30	69.3	5×10^4	−5	121	82.0	119	88.5	.927	
				129	84.6	127	91.9	.921	.9247
				136	87.0	132	94.6	.921	
				145	90.0	141	97.0	.928	
20	46.2	5×10^4	−9	75	63.9	157	102.6	.623	
				79	65.4	165	105.6	.620	.6233
				89	69.5	183	112.0	.621	
				99	73.7	199	117.2	.629	
30	69.3	5×10^4	−9	167	96.8	162	104.4	.927	
				173	98.4	169	107.1	.920	.9240
				184	101.4	176	109.5	.926	
				199	105.3	189	114.2	.923	
20	46.2	5×10^4	−12	80	65.7	166	106.0	.620	
				91	70.5	185	112.7	.626	.6255
				102	74.9	204	119.7	.626	
				113	79.2	222	126.1	.630	
30	69.3	5×10^4	−12	170	97.7	164	105.3	.927	
				143	89.3	138	95.9	.931	.9273
				154	92.9	150	100.2	.931	
				166	96.6	160	103.8	.920	

TABLE V. (*continued*).

Condenser No. I., Ice Dielectric, Readings.

Magneto.		r	T	No. 1.		No. 2.		E_1/E_2	Mean.
Volts.	Freq.			Defl.	E	Defl.	E		
20	46.2	5×10^4	−15	85	67.9	175	109.2	.623	
				95	72.1	193	115.7	.623	.6225
				102	74.0	205	120.0	.617	
				111	78.5	220	125.4	.627	
34	78.5	5×10^4	−15	163	95.7	125	91.1	1.050	
				176	99.3	135	94.6	1.049	1.0488
				190	103.0	144	98.0	1.050	
				203	106.2	154	101.7	1.046	
20	46.2	5×10^4	−18	81	66.3	170	107.3	.618	
				91	70.5	186	113.1	.623	.6217
				101	74.5	204	119.7	.622	
				111	78.3	220	125.3	.624	
34	78.5	5×10^4	−18	163	95.7	126	91.6	1.044	
				178	100.4	136	95.2	1.057	1.0497
				193	103.6	146	98.8	1.048	
				208	107.4	156	102.4	1.050	

air insulted at all points except where walls were pierced, and led through glass tubes at such places. The two wires averaged 12 inches apart. Their capacity, which must be subtracted from the calculated capacity in each case where they are used, was found from c_2, by the same method as the latter from $C+c_2$. The value found was 1.72×10^{-11} farad.

TABLE VI.

Results, Condenser No. I.

T	Eq. (2). $C+c_2$	Diel. Const.	
		C	K
− 2	4.140×10^{-8}	4.133×10^{-8}	93.9
− 3	4.149	4.142	94.3
− 5	4.124	4.117	93.6
− 9	4.204	4.197	95.3
−12	4.222	4.215	95.6
−15	4.232	4.225	96.0
−18	4.241	4.234	96.4

Ice Capacity.—Table V. gives the readings taken at various temperatures on this condenser, with ice as dielectric, with the resulting ratio E_1/E_2. The calculated values of $C+c_2$ and the dielectric constant, appear in Table VI. It was possible to keep the refrigerating room at each of the temperatures given, continuously, for unlimited time: and as the readings at any temperature required about fifteen minutes, and were taken after keeping the room at the desired temperature for an hour—the condenser temperature being read on a thermometer in close contact with the outside cylinder—it is probable that the dielectric, at the time of reading, was at very nearly the indicated temperature.

2. *Condenser No. II.*

Air Capacity.—Table VII. gives the data taken on condenser no. II. with air dielectric, with the calculated values of its capacity plus that of electrometer no. 2. Its true capacity was accordingly 2.219×10^{-10} farad.

TABLE VII.

Air Capacity, Condenser No. II.

Magneto.		r	No. 1.		No. 2.		E_1/E_2	Eqn. (1). $C+c_2$
Volts.	Freq.		Defl.	E	Defl.	E		
32	73.9	3×10^6	62	58.4	305	151.7	.386	
			58	56.5	290	146.8	.385	2.757×10^{-10}
			54	54.2	277	142.5	.381	
35	80.9	3×10^6	71	62.2	297	149.1	.416	
			68	61.0	288	146.2	.417	2.737×10^{-10}
			62	58.4	267	139.3	.418	

Mean, 2.747×10^{-10}

Ice Capacity.—Table VIII. gives the results of a run on this condenser at $-9°$ C., also at the temperature of liquid air, with the resulting dielectric constants.

TABLE VIII.

Results, Condenser No. II.

Magneto.		ν	No. 1.		No. 2.		E_1/E_2	$C+c_2$	C	K	T
Volts.	Freq.		Defl.	E	Defl.	E					
20	46.2	1×10^5	124	83.1	164	105.2	.790	2.117 × 10^{-8}	2.110 × 10^{-8}	95.2	−9
			134	87.0	179	110.7	.786				
			144	89.2	193	115.7	.776				
			154	92.7	206	120.3	.770				
30	69.3	1×10^5	151	91.7	116	87.4	1.049				−9
			134	87.0	106	83.3	1.043				
			115	81.3	96	79.0	1.029				
			106	76.8	86	74.4	1.032				
20	46.2	3×10^6	82	66.8	190	114.5	.583	7.185 × 10^{-10}	7.132 × 10^{-10}	3.0	−192
			92	70.8	210	121.8	.582				
			102	75.0	229	128.6	.583				
			112	78.9	248	135.4	.583				
30	69.3	3×10^6	197	104.8	192	115.2	.910				−192
			185	101.7	182	111.8	.910				
			173	98.5	172	108.1	.910				
			162	95.3	162	104.5	.912				

3. *Condenser No. III.*

TABLE IX.

Air Capacity, Condenser No. III.

Magneto.		ν	No. 1.		No. 2.		E_1/E_2	Eqn. (1). $C+c_2$
Volts.	Freq.		Defl.	E	Defl.	E		
34	78.5	3×10^6	40	47.3	256	135.6	.349	2.301×10^{-10}
			44	49.6	276	142.2	.349	
			47	51.3	291	147.1	.349	
			49	52.3	301	150.4	.347	

TABLE X.

Ice Capacity, Condenser No. III.

Magneto.		ν	T	No. 1.		No. 2.		E_1/E_2	Eqn. (2).		K
Volts.	Freq.			Defl.	E	Defl.	E		$C+c_2$	C	
20	46.2	1.5×10^5	−8	72	64.5	125	91.2	.708	1.708 × 10^{-8}	1.703 × 10^{-8}	96.1
				87	68.6	143	97.6	.704			
				99	73.8	161	104.1	.707			
				109	77.7	175	109.3	.710			
30	69.3	1.5×10^5	−8	226	111.8	153	102.3	1.093			
				206	107.2	144	98.1	1.087			
				190	103.1	134	94.4	1.095			
				175	99.2	124	90.7	1.089			

Air Capacity.—This is found from Table IX. to be 1.773×10^{-10} farad. Its capacity at $-8°$ C., with the calculated dielectric constant, appears in Table X.

Note.—In all reductions on condensers with ice dielectric, except those at liquid air temperature, the values given for $C+c_2$ include the capacity of the leads to the refrigerating room.

DISCUSSION OF RESULTS.

From the work on condenser no. I., whose capacity in air was large enough to measure with the same accuracy as in ice, it appears that for the frequency used, the dielectric constant of ice increases from $-2°$ to $-18°$ C., which was as low as the refrigerating room could be carried. The agreement between the constant determined by the small cylindrical condenser, that by the plate condenser, and that by no. I., seems to indicate that the thickness of ice used does not appreciably affect the constant, nor does the form of condenser. It is argued from this, that the ice was either under no great strain in either case, or that such strains were similar in all the condensers, which seems highly improbable, as the forms of condenser used differed so widely. The value obtained for liquid air—3.0—agrees exactly with the result given by Dewar and Fleming.[1]

It is believed that the results indicate a true maximum constant between $-18°$ and $-130°$ C. Also, that the constant with infinite wave-length is considerably larger than any work up to the present time seems to indicate. Zeleny and Andrews[2] have called attention to the possible enormous increase of dielectric constant, when the total absorbable charge is considered: it seems at least possible that absorption may completely explain the apparently large effect of frequency. This effect has been found by every observer who has worked on this determination, and is in all probability truly present. Work is now under way, with the purpose of determining the ratio of the total to the "free" charge, for ice.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
June, 1910.

[1] *Loc. cit.*

[2] Zeleny and Andrews, PHYS. REV., vol. 27, p. 65, 1908.

THE ROTATORY POWER OF QUARTZ, CINNOBAR, AND NICOTINE AT LOW TEMPERATURES.

By F. A. Molby.

THE writer has made a large number of measurements of the rotatory power of quartz plates cut perpendicular to the optic axis, using both right quartz and left quartz, through a range of temperatures from 25° C. to −190°. Measurements made with sodium light were published in the Physical Review, January, 1909. An abstract of later measurements with several wave-lengths of light was published in the Physical Review, February, 1910. The later measurements indicated that the temperature effect is slightly larger for the shorter wave-lengths of the visible spectrum than for the longer wave-lengths. The writer then decided to repeat the work, using stronger light sources and a larger number of wave-lengths, together with greater precaution against errors of observation. The result of the repeated work leads to the same conclusion, but the temperature effect is so nearly the same for all parts of the visible spectrum that very refined measurements must be made in order to thoroughly establish the statement that such a difference exists. A description of the apparatus used and a summary of the measurements, together with graphical representation of results, are given in this paper.

1. The Polariscope.

The instrument used was purchased from the makers, Schmidt and Haensch, for the writer's use in this work. It is of the Lippich type with two small polarizing nicols fixed in place, while the large polarizing nicol admits of adjustment for greater or smaller sensibility of the instrument, fixed however during a set of measurements. The framework is of one solid cast piece and the workmanship of mounting the nicols is such that it seems impossible that temperature or other strains could produce a shift

of the zero points so great as to be appreciable in this work. The parts of the instrument are not to any extent cooled by the cooling devices, and furthermore the work has been done in an inside room where the greatest range of temperature did not exceed four degrees during the winter and spring. The circle is graduated to hundredths of a degree and is fitted with two verniers and a slow movement tangent screw. The writer has been obliged to assume that the zero point remains fixed, as it would be very difficult to make alternate measurement of rotations and of zero point. It may be stated however that, for the most part, the measurements made at room temperature were followed on the same day by the measurements at the low temperature, so that errors on the zero reading should be the same for the one as for the other temperature.

2. QUARTZ HOLDER AND COOLER.

The box in which the quartz was placed for cooling was described in the PHYSICAL REVIEW, January, 1909. The glass tubes, *T* and *T* in the former description, were fastened in line by metal Y's and securely held in a box provided for packing wool around the brass box and cooler. This latter box was clamped to the frame of the polariscope and provided with tangent screws against the frame in such a way that sidewise and vertical tilting adjustments could be made. A silvered mirror was then put in the holder in the place of the quartz and a Gaussian eyepiece method used for adjusting the mirror until an image of the diaphragm in the eyepiece of the polariscope was reflected back upon itself. The clamping arrangement would then not admit of shifting; by several trials, removing the mirror and replacing it, the writer was convinced that the quartz would occupy the same position that the mirror occupied. By this arrangement the light path through the glass parts of the holder would be constant except for the quartz, which was removed when determinations were made of the zero readings of the instrument. The writer has assumed that no activity in the glass parts would result from cooling, but as *L* quartz and *R* quartz of almost equal thickness have been measured and the mean value for the result used, such small errors would, if always the same, cancel themselves out.

3. Temperature Measurements.

A new Leeds and Northrup potentiometer was used with a single iron-constantan thermal couple. The smallest division on the instrument being 5 microvolts and the space such that microvolts could be estimated, temperatures could be measured to 0.1 of a degree, but this accuracy is somewhat greater than can be claimed for the temperatures far removed from the points of calibration. The copper ring was inserted in the cold end of the couple as was formerly done and the couple was calibrated with the potentiometer by careful measurements using liquid oxygen and ice, CO_2 ether paste and ice, ice and a Reichsanstalt standard thermometer at room temperature, and ice against steam. In Table I. are given the values from which the following equation was derived and which fits very accurately intermediate temperatures measured with the standard thermometer.

$$E = 52.677\,T + .0288\,T^2 - .00013222\,T^3.$$

Table I.

T	− 183°.0	− 78°.2	21°.8	99°.2	Observed values.	
E	−7865	−3880	1161	5380	Observed values.	
T	− 188°.0	− 178°.0	− 173°.0	− 153°.0	− 123°.0	Computed values.
E	−8007	−7718	−7566	−6912	−5798	Computed values.

T is expressed in centigrade degrees and E is expressed in microvolts. The values for E were computed at five-degree intervals and a curve plotted on a large scale so that the temperature could be read with certainty to the nearest tenth of a degree as given by the calibration.

4. Monochromatic Light Sources.

A good spectrophotometer was adapted to the needs of the experiment by interchanging the collimator tube for the telescope tube so that the collimator could be shifted in azimuth while the telescope remained trained upon the polarizer of the polariscope. The Lummer-Brodhun cube was of course removed. A prism with about four-degree dispersion of the visible spectrum was used with the slit of the collimator at 0.8 mm.; to effect a better purification of the light color screens were used in the case of some wave-

lengths. The oxy-hydrogen flame was used for lithium wave-length 670.8 and for sodium 589.3. The mercury vapor lamp was used for wave-lengths 546.1, 435.9 and 404.7. A silver cadmium alloy was used in an electric arc as described by Mr. T. M. Lowary.[1] In this case the terminals of the arc were mounted each at an angle of forty-five degrees with the vertical and rotated at equal speeds in opposite directions.

The writer found that with terminals .25 inch in diameter held by set screws into copper rods .5 inch in diameter the arc could be used at an expense of about one cent per hour for the silver and with a current of ten amperes. The arc length is approximately two millimeters in length but the rotation makes it loop itself into a much larger area than would be obtained by stationary terminals. The writer found it quite satisfactory and not so troublesome to operate as is the oxyhydrogen flame. There are in it at least four very intense lines easy of separation from the other lines. From it the lines 467, 480.0, 508.6, 520.9 and the line 643.8 were used.

5. QUARTZES USED.

The quartz used was procured from the firm of Steeg and Reuter. Two of the plates were found to be of thickness so uniform that no difference could be detected with a micrometer, and two were found to give readings varying as much as .005 mm. from the mean reading. As calipered by Mr. H. G. Dorsey in the same way that he calipered his specimens for the low temperature work,[2] the mean of ten measurements on each quartz was as given below:

L, No. 1, 3.7843 mm.
L, No. 2, 2.8647 mm.
R, No. 1, 3.7494 mm.
R, No. 2, 2.8601 mm.
Sum of *L*'s 6.6490 mm.
Sum of *R*'s 6.6095 mm.

The two *L* quartzes were measured together in the polariscope work, likewise the two *R* quartzes. The instrument being in adjustment as already described, the zero reading was determined

[1]Standard Wave-Lengths for Spectroscopy and Polarimetry, Phil. Mag., August, 1909.

[2]PHYS. REV., vol. 25, August, 1907.

TABLE II.

Wave.	Qtz.	N.	Temp.	$(E)10^{-5}$.	Obs'd R.	R. at Std. T.	dR	R. mm.
404.7	R.	20	23.3	124.0	322.97	322.82	7.71	48.842
	R,	20	−185.0	−792.0	315.18	315.11		47.675
	L.	24	22.1	117.5	324.77	324.67	7.97	48.830
	L.	22	−186.6	−796.5	316.73	316.70		47.631
					Mean per cent. decrease 2.422			
435.9	R.	10	24.4	130.0	274.08	273.91	6.43	41.441
	R.	6	−138.2	−638.0	268.61			
	R.	10	−175.4	−764.0	267.76			
	R.	12	−188.8	−803.0	267.47	267.48		40.469
	L.	12	23.8	126.8	275.44	275.29	6.49	41.403
	L.	12	−187.0	−798.0	268.82	268.80		40.427
					Mean per cent. decrease 2.353			
467.0	R.	7	24.4	129.8	234.98	234.83	5.51	35.529
	R.	16	−188.6	−802.4	229.31	229.32		34.696
	L.	9	23.0	122.6	235.99	235.88	5.57	35.477
	L.	10	−188.0	−800.8	230.31	230.31		34.638
					Mean per cent. decrease 2.354			
480.0	R.	7	24.3	129.6	222.17	222.03	5.12	33.593
	R.	8	−187.8	−800.0	216.91	216.91		32.818
	L.	10	23.2	123.2	223.38	223.28	5.31	33.580
	L	9	−188.6	−802.3	217.96	217.97		32.782
					Mean per cent. decrease 2.344			
508.6	R.	7	24.3	129.6	196.13	196.01	4.45	29.656
	R.	10	−187.7	−800.0	191.56	191.56		28.983
	L.	8	23.4	124.2	197.07	196.98	4.63	29.625
	L.	7	−117.1	−556.0	193.53			
	L.	7	−155.4	−699.5	192.85			
	L.	7	−178.0	−772.0	192.49			
	L.	9	−189.0	−803.3	192.34	192.35		28.930
					Mean per cent. decrease 2.310			
520.9	L.	8	23.3	123.8	187.28	187.19	4.34	28.153
	L.	6	−116.1	−552.0	183.97			
	L.	7	−154.1	−695.0	183.32			
	L.	8	−178.0	−772.0	182.96			
	L.	10	−189.0	−803.3	182.84	182.85		27.501
					Mean per cent. decrease 2.319			
546.1	R.	12	24.5	130.2	168.47	168.36	3.88	25.472
	R.	6	−177.2	−769.5	164.60			
	R.	10	−189.0	−803.5	164.47	164.48		24.885
	L.	9	23.9	127.2	169.43	169.33	3.98	25.467
	L.	10	−187.3	−798.7	165.36	165.35		24.868
					Mean per cent. decrease 2.328			

TABLE II.—*Continued.*

Wave.	Qtz.	N.	Temp.	$(E)10^{-6}$.	Obs'd R.	R. at Std. T.	dR	R. mm.
589.3	R.	7	24.4	129.8	143.43	143.34	3.34	21.686
	R.	12	−186.4	−796.0	140.06	140.04		21.188
	L.	8	23.8	126.7	144.14	144.06	3.31	21.667
	L.	14	−187.4	−799.0	140.76	140.75		21.169
					Mean per cent. decrease 2.314			
643.8	R.	9	24.2	129.0	119.06	118.99	2.64	18.003
	R.	8	−187.8	−800.1	116.35	116.35		17.603
	L.	10	23.6	125.8	119.72	119.66	2.90	17.997
	L.	10	−187.4	−799.0	116.77	116.76		17.561
					Mean per cent. decrease 2.329			
670.8	R.	12	22.6	120.0	109.05	109.01	2.30	16.493
	R.	12	−186.9	−797.5	106.72	106.71		16.145
	L.	20	19.8	105.0	109.63	109.63	2.25	16.488
	L.	20	−185.3	−793.0	107.40	107.38		16.150
					Mean per cent. decrease 2.086			

Mean per cent. decrease from 20° to −188° C. (omitting the first and the last wave-length) is 2.33 per cent. of "R." at 20.0° C.

and the quartz put in place. For the most part the work at room temperature was followed on the same day by the work at low temperature and in all cases the quartz was not disturbed in its position from the time the first measurements were begun until the last were made so that the changes of rotation observed should be due entirely to temperature changes.

6. DATA.

With all wave-lengths but 404.7 and 670.8 a polarizing angle of 2°.5 was used. The results are summarized in Table II. Under the column "N" are given the number of settings made for the mean value of the rotation given opposite in the column "Observed Rotation." Under the column "Temp." are given the temperatures, read from the calibration of the thermal couple for the corresponding observed thermal E.M.F. Under the column "R. at Std. T." are given the rotations corrected to 20° C. and −188° C. respectively. Under the column "dR" are given the respective decreases in rotation when cooled from 20° C. to −188° C. Under the column "R. per mm." are given the specific rotations at the

standard temperatures, derived by dividing the "R. at Std. T." by the thickness of the quartz at room temperature. The per cent. decrease for each wave-length is given, *i. e.*, values derived by dividing "dR" by the corresponding rotations at 20° C. The room temperature coefficient of rotation as determined by Bates[1] was used for reduction to 20°. A mean value of the coefficient at −188° was found from the writer's work and used in reducing the rotations at liquid air temperature to the value which should be obtained at −188°. It will be seen that the specific rotation found for the R quartz is in each case slightly greater than that found for the L quartz. The accuracy of the observations is put to a rigorous test by the graphical methods to be described.

7. Curves.

In Fig. 1 the upper curve is plotted with observed values of rotation for the different wave-lengths as abscissæ and the amount by which the cooling decreased the rotation as ordinates. The

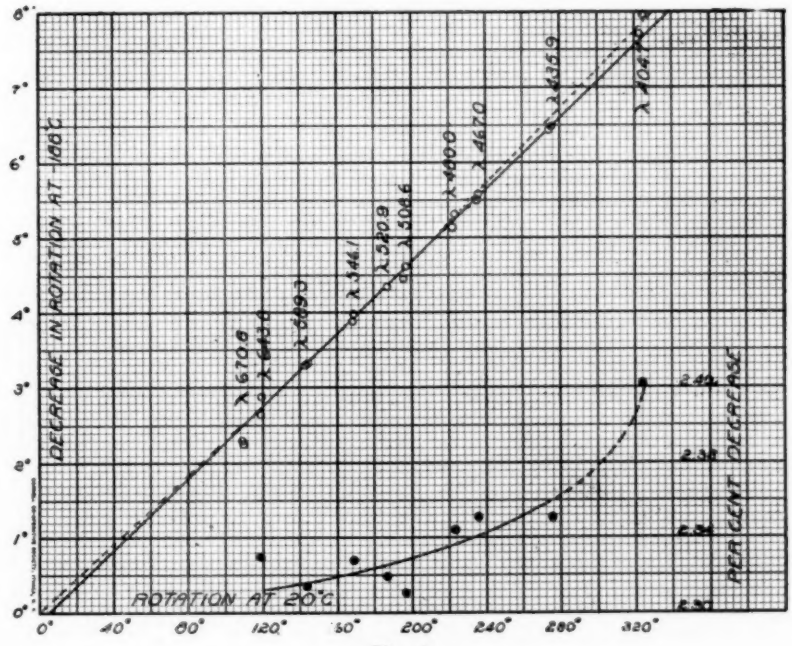

Fig. 1.

[1]Bulletin of the Bureau of Standards, Vol. II., No. II., pp. 239–248.

L quartz being a little thicker than the *R* quartz, its rotations are just enough greater to admit of plotting two points to represent the work on the two for any given wave-length. The points for the wave-lengths 404.7 and 670.8 may perhaps be disregarded for the reason that the sensibility is very much smaller in measurements at the end of the spectrum. The mean value of the points for the remaining number of wave-lengths are seen to form a straight line which could hardly be passed through the origin of coördinates. This in itself indicates that the temperature effect is relatively smaller for the long wave-lengths than for the short ones. Fig. 1 shows in the lower curve the same thing in a different way. There the ratio of dR divided by "R. at 20°," for each wave-length is plotted to the fourth significant figure of the ratio. This shows the work up in as bad a light as could well be done, and at the same time shows that as a group the shorter wave-lengths are affected most by the cooling. The difference is so small that if we take percentages from the lower curve as it is drawn, 2.36 per cent. at wave-length 435.9, and 2.316 per cent. at wave-length 589.3, the difference in the decrease for a 280-degree rotation would be 0°.12 as obtained by using the one or the other percentage. A numerical average of the percentages obtained with the eight wave-lengths, 435.9 to 643.8 inclusive, is a 2.33 per cent. decrease. The mean of the values for the point at wave-length 508.2 deviates from the straight line more than the others; the observed value corrected to −188° was 191.955, while a 2.33 per cent. decrease would have made it 191.917, which disagrees with the observed value by less than four parts in twenty thousand. It seems likely, therefore, that an equation

$$(R)_{-188} = (.9767)(R)_{20°}$$

is correct to about one part in 10,000.

Becquerel[1] found by a photographic method that the decrease in rotation for quartz from room to liquid air temperature is for different wave-lengths nearly proportional to the corresponding rotations and also states that it is 1/43, which is equal to 2.326 per cent.

[1]Sur le pouvoir rotatoire aux basses temperatures et sur la liaison entre l'absorption de la lumière et la polarisation rotatoire dans les cristaux de cinabre: Comtes Rendus, Dec. 14, 1908.

For sodium light he gives $R = 21°.71$ and $= 21°.20$ per mm. at the two respective temperatures. For the same wave-length the writer obtained a decrease of °.498 per mm. between 20°.0 and −188°.0 C.

The temperature coefficient at liquid air temperature has been derived from values of rotation given in Table II. by a straight line relation between temperature and rotation at that temperature. For R quartz between −175°.4 C. and −188°.0 the coefficient is

$$a = \frac{0.29}{(13.4)(273.91)} = .000079,$$

for wave-length 435.9. Similarly L quartz between −189 and −178 gives $a = .000069$ for the wave-length 508.6, and $a = .000060$ for wave-length 520.9, and R quartz between −189 and −177.2

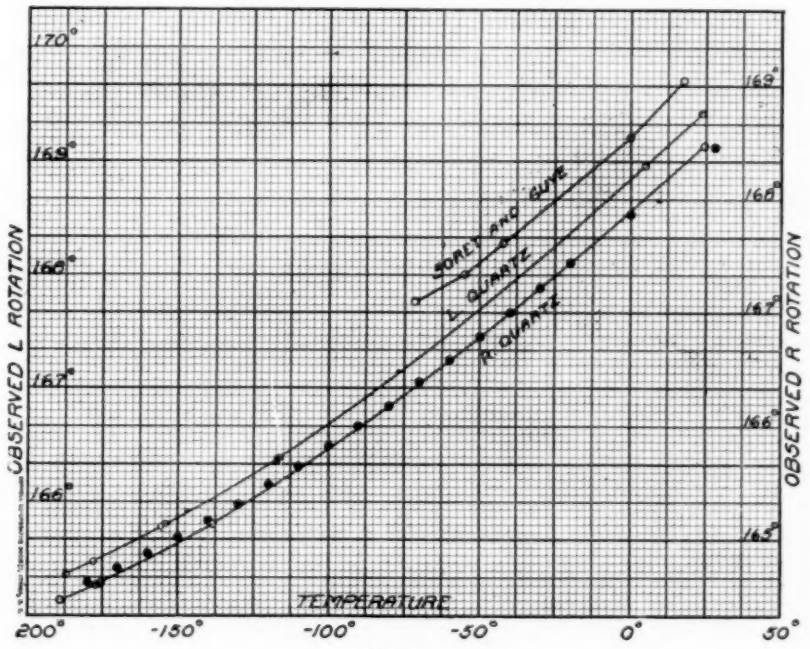

Fig. 2.

gives $a = .000065$ for wave-length 546.1. The mean of these four values gives $a = .00007$ at liquid air temperatures.

In Becquerel's article no mention is made of the rotation at temperatures between 20° and −188°. The writer showed graphi-

cally in the first article of this series the results obtained for the specific rotation of quartz through the range of temperature covered in this paper but with only sodium wave-length. Since the rotatory power for the several wave-lengths is decreased almost the same percentage, the rotations observed at an intermediate temperature should be proportional to the rotations at the standard temperatures for the respective wave-lengths. The wave-length 546.1 has been used as a standard wave-length and the several intermediate temperature observations from all the wave-lengths, Table II., are plotted so on the two curves in Fig. 2.

Fig. 2 shows the way in which the rotation changes with temperature through the 210° range, the observations for the two quartzes being plotted separately. The full black points on the *R* quartz curve are from data taken with a less accurate measurement but with slowly drifting temperature extending over a period of six hours' time. Each point represents the mean value from about seven settings of the polariscope and as many readings of the thermal E.M.F. The points on the Soret and Guye curve are reduced to values for my L quartz by multiplying their values of rotation per mm. by the thickness of my *L* quartz and by the ratio

$$\frac{[R]_\lambda = 546.1}{[R]_\lambda = 589.3} = 1.175166,$$

as given by Bates.[1] Soret and Guye give the following values for sodium light.

T	$-71°.5$	$-55°.3$	$-42°.3$	$0°.0$	$17°.7$
$[R]_D$	$21°.47$	$21°.505$	$21°.537$	$21°.654$	$21°.719$

Bates found for the temperature coefficient of rotation a value $a = .000144$ between 4° and 50° C. I have in Fig. 3 plotted the figures which he gives[2] as a summary of his measurements and find that his values cannot be said to give a straight line unless we choose to ignore the points at 16°.049 C. and 20°.761 C., two temperatures at which the greatest accuracy of measurement would be expected.

In Fig. 3, lower curve, I have plotted my observations together with values that would have been observed at temperatures up

[1] Ibid.

[2] Bulletin of the Bureau of Standards, Vol. II., No. II., p. 246. Table II

to 840° based upon the coefficient given by Joubert for the high temperatures.[1] The one full black circle is a point computed for temp. 570° by the use of a cubic equation derived by LeChatelier.[2]

The curves in Fig. 2 and in Fig. 3 show that there is a gradual change in the rotation and also in the rate of change of the rota-

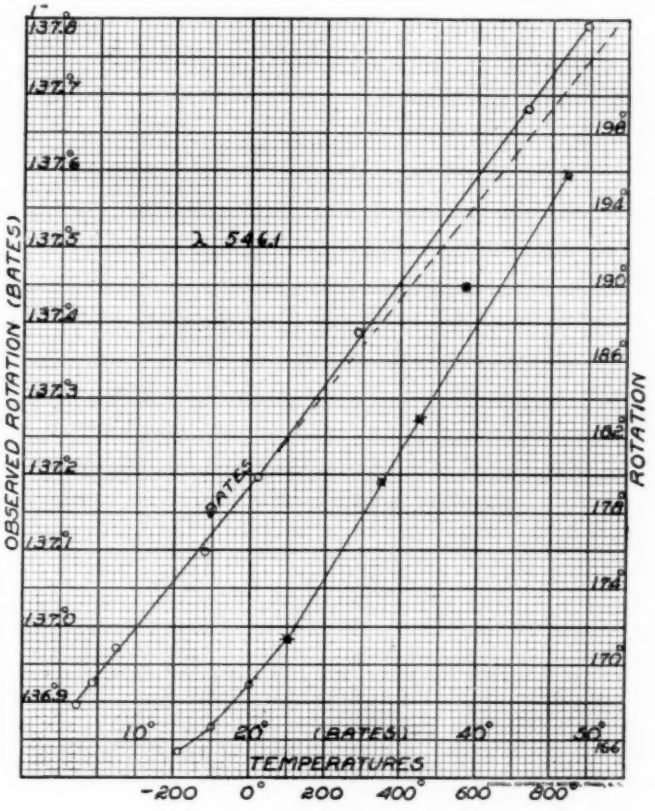

Fig. 3.

tory power as the temperature is increased from −190° to 840°. The largest value given by Joubert for the temperature coefficient is $a = .000190$ between 0° and 840° C. The next is $a = .000186$ between 0° and 448°. From these two values the mean temperature coefficient between 448° and 840° is .000194; from these sev-

[1]See Landolt and Bornstein, Myerhoffer Tabellen, 5 Auflage, p. 707, also Beiblätter, 2, pp. 657–8, year 1878.

[2]Landolt and Bornstein, Myerhoffer Tabellen, 5 Auflage.

eral values then the temperature coefficient of rotation at room temperature appears to be twice as great as the temperature coefficient at $-190°$, and that at 600° or 700° C. is nearly three times as great as that at $-190°$.

Cinnabar: Rotation between Room and Liquid Air Temperatures.—A plate of cinnabar 0.35 mm. thick and 2×2.5 mm. in size was procured from the firm of Steeg and Reuter. This was used in the same way that the quartz was used for polariscopic measurements. The degree of accuracy in measuring the rotation was not to be compared with the possible accuracy in the work with quartz, but the values obtained are at least consistent with each other within 0.5 per cent. The specimen was completely opaque to sodium light at room temperature; but when cooled to liquid air temperature it was sufficiently transparent that measurements were made with the sodium flame of oxyhydrogen. Rotations were measured at room and at liquid air temperature for the lithium oxyhydrogen flame. Measurements were made at several temperatures with the cadmium line 643.8 from the rotating arc. The results are given in Table III. As in Table II., *N* indicates the number of settings of the polariscope from which the value of "Obs'd R." was obtained at each of the given temperatures.

TABLE III.

λ	*N*	*T*	Obs'd Rot.	λ	*N*	*T*	Obs'd Rot.
643.8	20	21.7	94.45	643.8	12	−161.2	81.89
"	14	4.0	92.85	"	16	−173.7	81.67
"	11	− 17.0	90.85	"	12	−186.3	81.28
"	13	− 46.4	88.42				
"	12	− 63.2	87.44	670.8	10	22.0	75.25
"	13	− 94.5	84.87	"	11	−184.5	66.50
"	12	−100.7	84.25				
"	13	−124.0	82.72	589.3	12	−187.4	142.0
"	12	−148.4	82.02				

Thickness of cinnabar plate 0.35 mm.

The results are shown graphically in Fig. 4. The curve shows a pretty uniform and rapid decrease in the rotation when cooled between 20° and $-125°$. From $-125°$ to $-188°$ the decrease in

rotation is very much less than that for a corresponding drop of temperature above $-125°$.

In the previously mentioned paper of Becquerel are some marginal values with the reproduction of his photograph which summarized are as follows: Thickness of cinnabar, 1.502 mm.

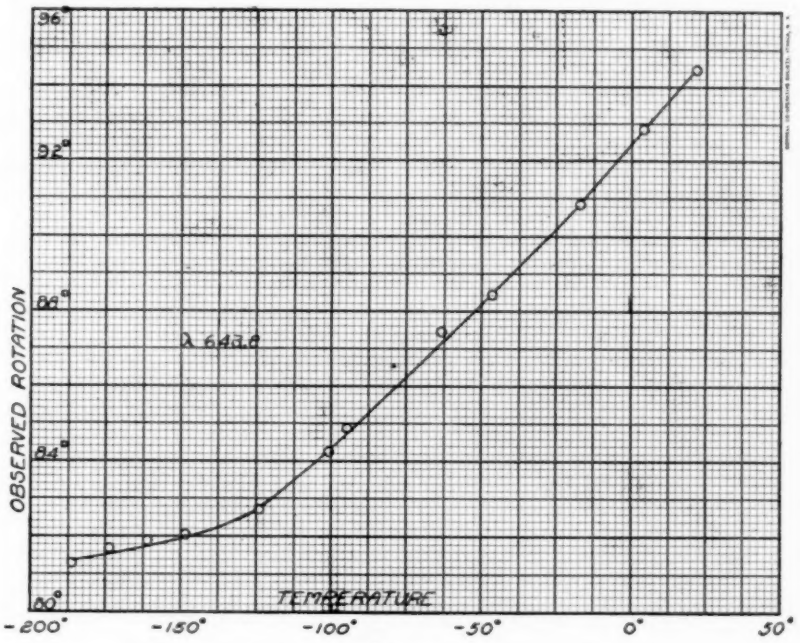

Fig. 4.

Temp.	18°	18°	−188°	−188°	−188°	−188°	−188°
λ	657.3	619.4	642.2	600.6	(579.5)[1]	567.2	559.9
Rot'n	3π	4π	3π	4π	5π	6π	7π

At 18° C. the absorption band extended to wave-length 619.4, but at −188 it did not extend so far as wave-length 559.9. By plotting dispersion curves from his data at the two temperatures I find that he obtained a decrease of about 10 per cent. for the rotation at wave-length 644.0 while the decrease is about 13 per cent. as obtained by the writer for the rotation at wave-length 643.8. Qualitatively the results agree but not quantitatively.

Rotatory Power of Nicotine between 20° C. and −90° C.—A

[1]Published as 569.5, apparently an error in printing.

sample of pure nicotine procured from Merck and Company was found to be unworkable at temperatures below 0° C. Through the kindness of Professor Orndorff the nicotine was very successfully redistilled by his assistant, Mr. A. W. Conklin. The nicotine was heated to 200° and held at that temperature for several hours, while an atmosphere of hydrogen, dried by passing through several powerful drying reagents, was passed over the liquid. The nico-

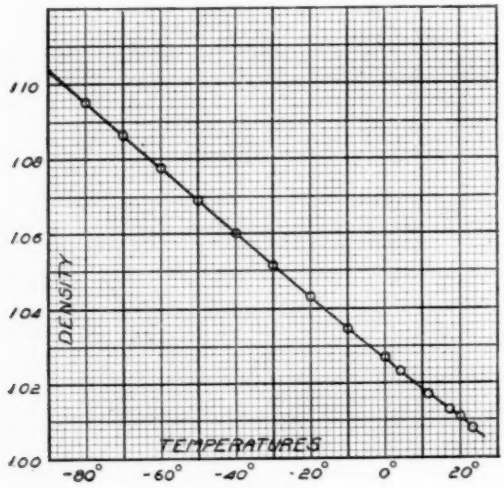

Fig. 5.

tine was then fractionally distilled and the better fraction used for the optical study. The density of the nicotine, determined after it had been used, was found in precisely the same manner as was described for finding the density of limonene[1] throughout the range of temperature studied. The density at 20° C. = 1.0107. A volume of 8.789 c.c. at 20° was found to occupy a volume of 8.111 c.c. at −80°. The change per degree temperature change is found to be slightly greater at the low temperature than at the ordinary temperature.

The relation between temperature and density is shown in Fig. 5 to be quite nearly linear. Densities were read from this curve for use in computing the specific rotation from the observed rotations at the several temperatures, an extrapolation being made for

[1] PHYS. REV., January, 1910, pp. 81–83.

the temperature −90°. A few of the points are given numerically in Table IV.

TABLE IV.

T	20.	0.	−20.	−40.	−60.	−80.
D	1.0107	1.027	1.043	1.060	1.0775	1.095 grams per c.c.

The method of observing rotations and temperatures was described in the PHYSICAL REVIEW, January, 1910, in an article on "The Rotatory Power of Limonene at Low Temperatures." The

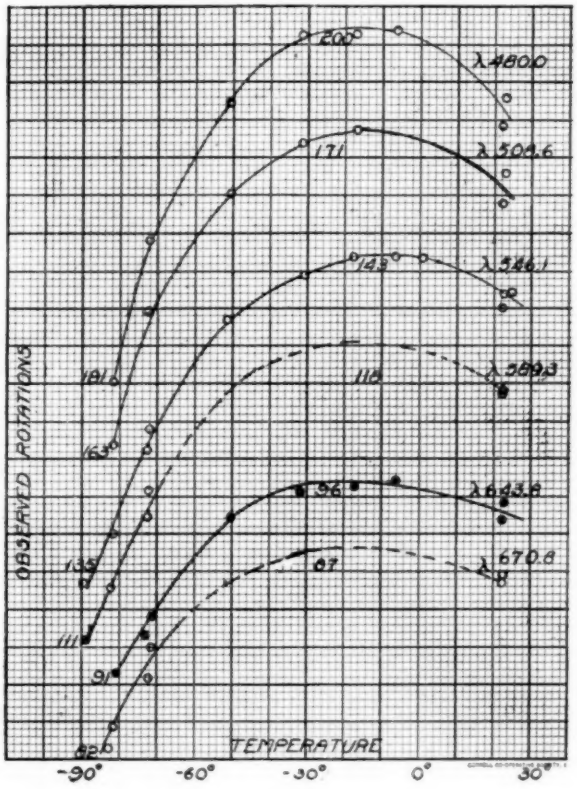

Fig. 6.

length of the liquid column used for measurement of rotatory power of nicotine was 7.79 cm. The same light sources were used and in the same manner as already described in the work on quartz. The short wave-lengths were not used because of a small amount

of oxidation which took place during the time in which preliminary measurements were being made and which made good work difficult in that end of the spectrum; there is with the freshly distilled product a very considerable absorption in the violet, as will be shown in a later article on nicotine. In Fig. 6 are curves showing the relation between *temperature* and *observed rotation* for the several wave-lengths; only two ordinates are numbered on each curve,

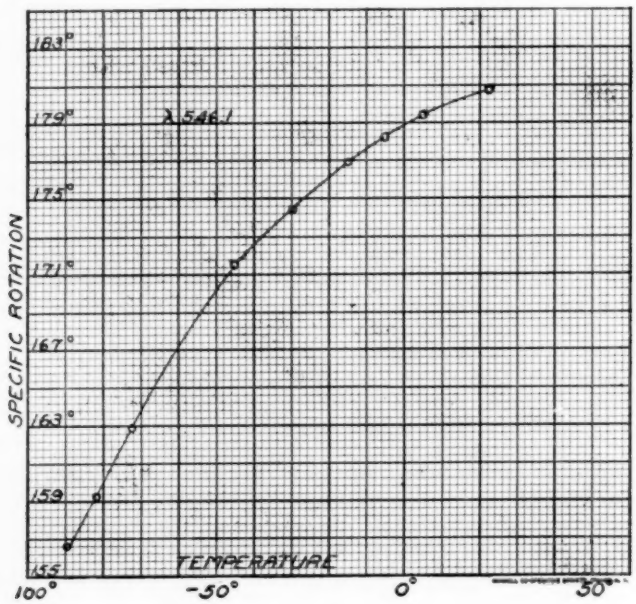

Fig. 7.

as for example with 480.0 the observed rotation was 191° at temp. −82°, while it was 200° at −34°. The maxima are not well defined, but seem to occur at about the same temperature. At room temperature the points show discrepancies, the lower ones in each case being made on a later date than the upper ones and doubtless show deterioration in purity of the specimen, although the tube was thought to be tightly closed by rubber tubing and a pinchcock. In Table V. are given the observed values for a few points, all of these measurements being made on the same day and therefore showing the true temperature effect as nearly as the writer

was able to measure it. The column "N" designates the number of settings made from which the mean value is given in the column "Obs'd R." opposite the temperature "T," at which they were made. The column $[R]_T^{\lambda}$ gives the specific rotation for the corresponding temperatures as obtained by the equation

$$[R]_T^{\lambda} = \frac{\text{Obs'd R.}}{L(D)}$$

in which L is the length of the liquid column expressed in decimeters and D is the density at the corresponding temperature.

TABLE V.

λ	N	T	Obs'd R.	$[R]_T^{\lambda}$
480.0	10	22°.3	197°.83	251.81
	7	−81 .5	191 .04	223.67
508.6	5	22 .4	169 .76	216.08
	4	−81 .5	163 .38	191.27
546.1	6	22 .4	141 .96	180.70
	8	−72 .5	138 .25	162.85
	8	−81 .5	136 .02	159.24
	8	−89 .7	134 .69	156.61
	[1]	−45 .0	142 .24	171.54
	[1]	−30 .0	142 .86	174.42
	[1]	−15 .0	143 .15	176.90
	[1]	− 5 .0	143 .10	178.20
	[1]	5 .0	142 .92	179.40
589.3	6	22 .7	117 .80	149.97
	6	−71 .5	114 .44	135.02
	7	−82 .1	112 .57	131.73
	14	−89 .0	111 .19	129.40
643.8	5	22 .5	95 .36	121.38
	9	−71 .2	92 .82	109.51
	5	−80 .7	91 .34	106.98
670.8	10	22 .5	86 .76	110.43
	8	−72 .2	84 .18	99.23
	9	−81 .6	82 .84	96.98

[1]Values from curves, Fig. 8.

Fig. 7 shows the relation between specific rotation and temperature through the range of temperature −90° to 24° as obtained from the rotations plotted in Fig. 6, wave-length 546.1. It is somewhat of a surprise to find that the appearance of the curve should be so very different when only the change in density of the

liquid is responsible for the disappearance of a maximum from the curve.

It is seen that the specific rotation decreases with decreasing temperature very much more rapidly at the low temperatures than at the ordinary temperature. The liquid was very viscous at $-90°$, this being the lowest temperature at which observations could be made with anything like the length of liquid column used. It would be interesting to study a thin layer to see what the effect

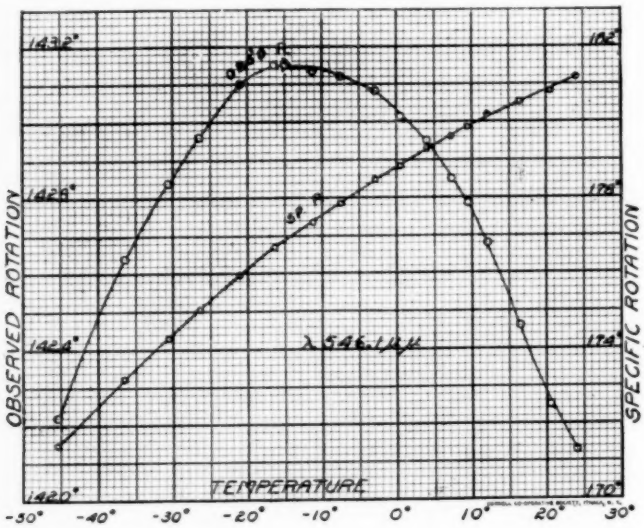

Fig. 8.

might be during the process of solidification. In this experiment solidification seemed to form globules which move about in the liquid and depolarize the light so that no settings could be obtained.

A series of observations were made with wave-length 546.1, while the temperature was gradually lowered from 20° to $-45°.3$, observations extending through a period of two and one half hours. The results are plotted in the curve in Fig. 8 and are plotted to such a scale as to show the temperature at which an *apparent maximum rotation* is reached.

Each point plotted is the mean value for five readings of both rotation and thermal E.M.F. The specific rotation at the same

temperatures was computed and the curve is plotted in Fig. 8 to show the contrast between observed rotation and specific rotation. The maximum observed rotation is at a temperature approximately $-15°$ C.

It is of interest to see graphically a comparison of relative effects of temperature upon the rotation of different wave-lengths. In

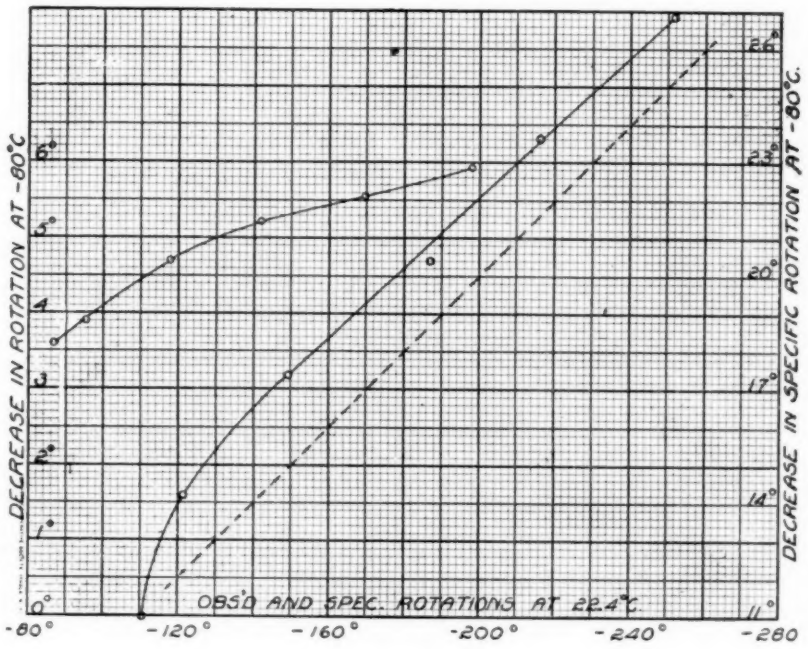

Fig. 9.

Fig. 9, upper curve, the rotations observed at 22°.4 C. are plotted as abscissæ and the decrease in the observed rotation at $-80°$ as ordinates, the latter being found by reference to the curves at the low temperature end. In the same figure, lower curve, the specific rotations at room temperature are plotted as abscissæ with the decrease in specific rotation, for the corresponding specific rotations, at $-80°$ C. as ordinates. The dotted line below the curve is a line which if extended would pass through (O, O) on the coördinate axes. From the measurements with wave-lengths at the central part of the spectrum it might therefore appear that the temperature effect is

relatively the same for all wave-lengths, but the measurements with the longer wave-lengths show greater relative decreases.

In a later paper the writer will give the results of spectrophotometric measurements made with the nicotine at low temperatures and if possible show a relation between the temperature effect upon absorption and the temperature effect upon the rotatory power of the substance. The measurements already made indicate a shift of the absorption band from the violet toward the blue, and at the same time an increase of transmission is noted in the red end of the spectrum when cooled to the low temperature. An unmistakable fluorescence, though not strong, is also noted from about wave-length 500$\mu\mu$ to the red end of the spectrum.

The writer desires to thank the several professors for their unfailing interest and for the privilege of using many valuable pieces of apparatus during the progress of the work. He wishes to acknowledge aid from a grant made by the Smithsonian Institution for investigations in heat, a part of which was applied on apparatus for this work. He is likewise appreciative for many favors done by members of the instructing staff in the departments of physics and chemistry.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
June, 1910.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

A DIRECT-READING CANDLE-POWER SCALE FOR PRECISION PHOTOMETERS.[1]

BY GEORGE W. MIDDLEKAUFF.

THE device described was designed for use in the precision work of the Bureau of Standards, the object being to eliminate the labor and possibility of errors of calculation when settings are read from a centimeter scale on the photometer bar.

The difficulty in using an ordinary candle-power scale for this class of work is that it is practically impossible to adjust the comparison lamp to fit the scale with the necessary accuracy, especially since it is desirable to use at least six standards in the adjustment.

The new scale avoids this difficulty by reversing the process, that is, it adapts itself perfectly to the adjustment of the apparatus. The scale proper is the rectangular figure *A BD C*, and is calculated on the basis that a 16-c.p. lamp is to measured with the photometer screen at a distance of approximately 120 cm. from the lamp. The sides *A B* and *CD* are two ordinary candle-power scales calculated to suit the arrangement that the comparison lamp moves with the photometer screen and remains at a constant distance from it. The oblique lines join corresponding points of graduation in the two scales, and hence any line drawn across them and parallel to the length of the rectangle will be divided in exactly the same proportion as *A B* and *CD* and will, therefore, be a candle-power scale also. Hence this rectangle includes an indefinite number of linear candle-power scales and corresponding to any fixed adjustment of the comparison lamp that will throw the 16-c.p. balance anywhere within the limits of the 16-c.p. oblique line there will be in the rectangle a linear candle-power scale that will exactly fit the adjustment of the comparison lamp, and therefore of the apparatus.

The scale as actually used in the Bureau is graduated into one-tenth candle divisions and is printed on sheets of paper about 35 cm. square, one of which is intended for each separate set of lamps photometered. While observations are made the sheet is wrapped on a cylinder placed immediately under

[1]Abstract of a paper presented at the Washington meeting of the Physical Society, April 23, 1910.

the photometer carriage, the line XX on the paper being at exactly 120 cm. from the lamp measured.

By means of a printing magnet suspended from the photometer carriage, the observer prints on the paper a record dot for each setting as it is made, the dots being kept separate by means of an electrically-driven clock which turns the cylinder a half millimeter after each dot is made. In this manner the settings on each lamp are represented by a group of record dots.

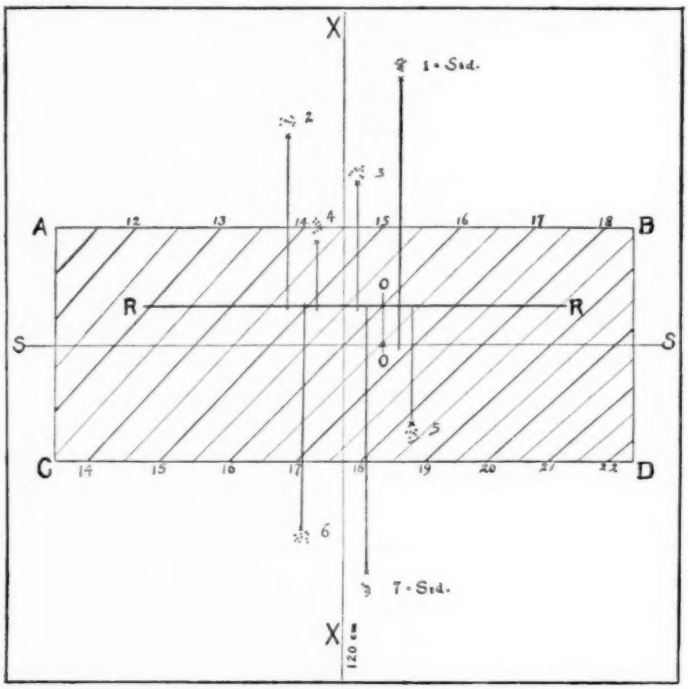

Fig. 6.

The measurements being completed, the center of gravity of each group of dots corresponding to the standards (as No. 1 and No. 7 in the figure) is read on the "Standard Scale" SS by means of a T-square and the mean of these readings is indicated by a mark on this scale. For the sake of clearness, suppose that this mean falls at 16.54 c.p. while its true value is known to be exactly 16.00 c.p. This shows that the scale that fits the record printed on the sheet lies above the standard scale. Its exact position is very easily and accurately determined by drawing a line OO parallel to XX through the mark made at 16.54 c.p. on SS and continuing it until it intersects the 16-c.p. oblique line. The line RR drawn through this point of intersection and parallel to SS is the linear candle-power scale desired, and from which the true values of all the unknown lamps (as No. 2 to No. 6) may be read

directly in candle-power to the nearest 0.01 of a candle. If from each group of dots a line is actually drawn parallel to *XX* and intersecting the "Record Scale" *RR*, the candle-power values are more accurately read and more easily checked by the second reader.

Lamps of intensities such as 32-c.p. and 8-c.p. may be balanced within the range of this scale by placing rotating sectored disks on the proper side of the photometer screen when the lamps are measured, the disk factors being then applied to the readings taken from the scale. For the 8-c.p. lamp it is better, however, for the sake of keeping the illumination of the screen constant, to continue the scale double its length toward zero, then divide it into two sections and move the 8-c.p. section forward placing it immediately below the 16-c.p. section. With the 8-c.p. lamp moved up on the bar by the same distance, its balance is made to fall near that of a 16-c.p. lamp without the use of a disk. Its value is read directly in candle-power from a record scale, *RR*, drawn in the same relative position on the 8-c.p. section as the corresponding scale is drawn on the 16-c.p., or standard, section.

By means of this device the results of a half day's work on the photometer can be read off in but a few minutes and the final record can be checked entirely by inspection. There are no tedious preliminary adjustments required and the photometer settings are made and recorded rapidly and accurately. The eye is not unnecessarily fatigued by alternate reading of photometer screen and scale and the observer is almost entirely freed from prejudice in making the settings. The record is simple, permanent and complete, and is easily read, checked and filed.

A Modified Method for the Determination of Relative Wave-Lengths, Especially Adapted to the Establishment of Secondary Spectroscopic Standards.[1]

By Irwin G. Priest.

THE purpose of this investigation was to develop and test a method for the determination of relative wave-lengths that would meet the present requirement for the establishment of secondary spectroscopic standards. This requirement, in the ideal, is the ability to determine the ratio of two wave-lengths with an uncertainty not greater than 1/4,000,000.

By the correlation of previously known phenomena and principles, a practical method has been formulated in which the effects of instrumental defects and the "personal equation" as well as the chances of accidental errors are reduced. This method is characterized by the following features:

1. Use of the *reflection* interference fringes of two parallel mirrors, one

[1]Condensed from a paper to be published in full in the Bulletin of the Bureau of Standards. Presented at the Washington meeting of the Physical Society, April 22–23, 1910.

of which is heavily silvered and the other partially, the light entering and leaving the interferometer through this partial mirror and in direction perpendicular to it.[1] By the action of a 20-mm. converging lens the fringe system is focused upon the wide-open slit of a spectrometer, so that the fringe systems of different wave-lengths can be observed at the spectrometer telescope.

2. Use of double *increment* of the distance between the two mirrors instead of the double *distance* as the difference of path.

3. Use of the method of flexure of elastic material to measure the fractional part of the order of interference. Springs and a lever are so arranged that the turning of a micrometer screw produces a displacement of the partial mirror, the ratio of the displacement of the screw to the displacement of the mirror being about 7,500/1. Readings of the micrometer head for the disappearance of certain bright spots at the centers of the fringe systems for the standard and unknown wave-lengths give the requisite data for evaluating the fractional part of the order of interference.

As an indication of the precision attainable with the method the following data were given: For seven determinations of the same wave-length, six of these depending each upon 10 measures of the fractional part of the order and one upon only five measures, the average departure from the mean was 1/8,400,000 while the maximum departure was 1/3,900,000. These determinations included all made on this line relative to red cadmium with an order of interference high enough to give this order of precision.

By measuring the previously known ratio of the red and green cadmium wave-lengths it was shown that the systematic error of the method could be made less than the accidental errors. The actual departures of the measured values of this ratio from the true values were 1/7,200,000, 1/19,000,000 and 1/3,100,000, the last being under unfavorable conditions that may be avoided.

The method is said to possess the following advantages over the "Method of Diameters,"[2] as it is now universally used in the establishment of spectroscopic standards.

1. The procedure being more direct and involving fewer assumptions, permits of a more rigorous control of the errors.

2. There is less chance of apparatus error such as imperfections of the mirrors, errors in adjustments, and aberrations of the optical system, the requirements of instrumental perfection being much less severe.

3. The chance of error due to distortion of the photographic image is eliminated.

[1] Hamy, Jour. de Phys., 5, pp. 789–809, 1906. Priest, Bull. Bur. Standards, 5, 483–484, 1909.

[2] Fabry and Buisson, Jour. de Phys. (4), 7, 169, 1908. Eversheim, Ann. der Phys., 30, 815, 1909. Pfund, Astrophy. Jour., 28, 197, 1908.

4. The error due to small temperature changes is eliminated without resorting to a thermostat.

5. There are fewer chances for accidental error, because, while in the "Method of Diameters," the precision of each determination is affected by the accidental errors of fourteen or eighteen scale readings, plus the number of scale readings involved in one or two determinations of focal length, the precision of each determination by the present method is affected by the accidental errors of only six scale readings; and by two of these in so slight a degree as to make the precision of the measured wave-length sensibly a function of the errors of only four scale readings.

6. The error due to dispersion of phase on reflection is automatically eliminated without further discussion or correction.

7. The "personal equation" in the measurement of the order of interference is eliminated.

On account of these advantages the author believes the method will give a closer approximation than has hitherto been made to the ideal accuracy of 1/4,000,000.

The method is now being used to determine the wave-lengths of neon, the lines of which are exceedingly well suited for spectroscopic standards on account of their homogeneity, brightness and convenience of production. Being well distributed through the red, orange and yellow where iron lines are scarce and unsatisfactory, they will fill a gap in the present series of standards. It is hoped to publish these values within a few months.

BUREAU OF STANDARDS,
WASHINGTON,
April, 1910.

THE SELECTIVE RADIATION OF PLATINUM.[1]

BY EDW. P. HYDE.

IN a pevious communication before the Physical Society the author described two new photometric methods for studying selective radiation from substances that can be mounted in exhausted bulbs. The results of the application of these methods to carbon in various forms, tantalum, tungsten, and osmium were given. Since that time data have been obtained on platinum which is of considerable interest in comparison with that obtained on other metals, since platinum has commonly served as a reference point.

It appears, as judged by the photometric criteria, that platinum is more

[1]Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–23, 1910.

selective than carbon in any of its commercial lamp forms, and less selective than tantalum, tungsten or osmium.

Although comparative measurements on an electrically heated black body were made prior to the previous communication, the results were not presented. They are referred to briefly in the present paper. As might have been expected, the values found for the black body are not very different from those found for untreated carbon, although there is some indication that untreated carbon is slightly selective.

In the present paper an interesting deduction from the Wien equation for spectral energy distribution is presented. Suppose that one substance, A, is at some arbitrary temperature, T_A, and a second substance, B, is brought to such a temperature, T_B, that the ratio of the emission $(J_{\lambda_1})_B$ at wave-length λ_1, to the emission $(J_{\lambda_2})_B$ at wave-length λ_2 for the substance B is the same as the corresponding ratio $(J_{\lambda_1})_A : (J_{\lambda_2})_A$ for substance A. Further let the temperature of A be increased to some arbitrary higher temperature T_A', and the temperature of B be increased to T_B', such that as before,

$$(J_{\lambda_1}')_A : (J_{\lambda_2}')_A = (J_{\lambda_1}')_B : (J_{\lambda_2}')_B .$$

If these two substances obey the generalized Wien equation (or Planck equation, if λ_1 and λ_2 are in the visible spectrum and the temperature is not too great, say 2500° absolute) it can be shown, on the basis of one or two assumptions which would seem to be well justified, that

$$(J_{\lambda_1}')_A : (J_{\lambda_1}')_B = (J_{\lambda_1})_A : (J_{\lambda_1})_B$$

provided the exponent α in the generalized Wien equation does not vary. Taking λ_1 and λ_2 in the visible spectrum, observations with the spectrophotometer, using the above criterion, indicate that α is quite constant for tantalum and tungsten as compared with carbon, but that it apparently undergoes a slight change for osmium.

These results combined with those published by Coblentz on the change in α with temperature would indicate that the variation in α takes place, not with change in temperature, but rather with change in wave-length, since the latter is involved in the method employed for determining α. Hence, it would appear that Wien's equation with constant α does not represent the distribution of energy in the spectra of the metals tantalum, tungsten and osmium. Whether or not Planck's equation in a generalized form will hold can be shown by the determination of α by Paschen's method, starting with the Planck, rather than with the Wien equation.

It is also shown that starting with either the Wien or the Planck equation the total emission (Stefan-Boltzmann Law) obtained by integration from $\lambda = 0$ to $\lambda = \infty$, is of such form in α that for ordinary temperatures a decrease in α produces an increase in the total emission, rather than a decrease. There

is therefore no ground for the statement sometimes made that a for metals must decrease at all temperatures in order that the total emission should not surpass that of a black body at the same temperature. Moreover, the rate of decrease in a as given by Coblentz is so great that on the assumption of the Stefan-Boltzmann Law the total emission for platinum at 1400° is only a very small fraction of the total emission at 1200°—a result which shows conclusively that there is some inconsistency in the given values of a for platinum. A similar conclusion is also evident from the values of a given for some of the other metals.

A Characteristic of Spectral Energy Curves.[1]

By W. W. Coblentz.

In a previous communication[2] under the above title the writer discussed the method used in computing the wave-length of maximum emission in the spectral energy curves of black body radiation. Attention was called to the fact that, by using the solution from the Wien equation, the λ_{max} obtained from the values of λ_1 and λ_2 taken far apart is larger than for λ_1 and λ_2 taken close to λ_{max}. A long and complicated correction factor was deduced, which reduced this uniform decrease to a fairly constant value of λ_m whatever the values of λ_1 and λ_2 and whatever the temperature. This correction factor has proven a delusion and a snare in spite of its applicability at all temperatures.

The object of the present communciation is to call attention to this fact, and to give some preliminary data based on a new series of fourteen spectral energy curves obtained with a vacuum bolometer (having a hemispherical mirror to "blacken" it), the computations of λ_{max} being made as described in the following paragraphs. The method of computation is due to Dr. E. Buckingham and will be fully described by him.

The value of λ_{max} cannot be obtained directly from the Planck equation; but a value can be obtained for it from the Wien equation, provided, for the same value of the emissivity, E, we can find a relation between the λ on these two curves. This relation is

$$(1) \qquad \lambda' = \lambda \left[1 + \frac{\lambda}{\frac{c_2}{T} - a\lambda} \left(\frac{1}{e^{c_2/\lambda T} - 1} \right) \right]$$

[1]Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–23, 1910.

[2]Phys. Rev., 29, p. 553, 1909.

where λ' refers to the point on the Wien curve, and λ to the point of same emissivity on the Plank curve. From the Wien equation

$$\lambda'_{max} = \frac{(\log \lambda_2' - \log \lambda_1')\lambda_1'\lambda_2'}{(\lambda_2' - \lambda_1') \log^e}, \tag{2}$$

and finally the wave-length of maximum emission λ_{max} on the Planck curve is obtained from the equation

$$\lambda_{max} = 1.00678\, \lambda'_{max}. \tag{3}$$

The difference between λ_1 and λ_1' (Planck-Wien curves) is negligible and we are concerned only with λ_2 and λ_2' on the long wave-length side of the spectral energy curve.

In practice it is assumed that the observed energy curve fits the Planck equation, and the observed values of λ_2 are substituted in eq. (1) from which λ_2' is then computed.

This value of λ_2' (with the observed $\lambda_1 = \lambda_1'$) is then substituted in eq. (2) which gives the value of λ'_{max} on the Wien curve. Finally the λ'_{max} (mean value for all vals. of λ_1 and λ_2 usually 6 to 8) is reduced back to the observed (Planck) curve by eq. (3). Now, if the observed curve fits the Planck equation, the various values of λ'_{max} must be the same within the limits of experimental errors.

In other words, the uniformly decreasing values of λ_{max} formerly observed must now be constant. This has been found true for a range of 700° to 1400° C. In Table I. is a sample selected at random, the values of λ_2 varying over an unusually wide range. In this table column 1 gives the value of λ_2 on the observed normal spectral energy curve; column 2 gives the corresponding value of λ_2' on the theoretical Wien curve; column 3 gives the value of λ_{max} by using the observed value of λ_2 (column 2) in the Wien equation (eq. 2); and column 4 gives the true value of $\lambda'_{max} = 1.95\mu$, as it would be found on the Wien curve from eq. (2). But the Wien curve is not the observed curve, so that finally the true value is (eq. 3) $\lambda_{max} = 1.0068\, \lambda'_{max} = 1.963\mu$. Since $T = 1{,}504$, $\lambda_{max} T = 2{,}950$ and $C_2 = 2{,}950 \times 4.965 = 14{,}600$.

The values in column 4, Table I., are as close as it is possible to read the values of λ_1 and λ_2 from the spectral energy curves. The tentative value of the mean fourteen energy curves is $\lambda_m T = 2{,}940$, which is about the same as observed by Lummer and Pringsheim, which is accidental, since they did not use the latest and best refractive indices of fluorite obtained by Paschen and by Langley and his assistants. In conclusion, it may be added that the aforesaid data seem to indicate a conclusive test of the validity of the Planck equation for the temperatures and spectral regions investigated.

TABLE I.

λ_2 (obsd.)	λ_2' (eq. 2)	λ_{max}	λ'_{max}
6.60μ	6.03μ	1.998μ	1.953μ
5.35	5.01	1.994	1.951
4.88	4.61	1.993	1.950
4.16	3.98	1.985	1.953
3.58	3.46	1.971	1.945
2.28	2.26	1.960	1.952
			Mean 1.950μ

ON THE SELECTIVE REFLECTION OF OXIDES.[1]

BY LEIGHTON B. MORSE.

AT present there are data[2] on the reflection of the crystalline oxides of six elements in the infra-red spectrum. Four of these selectively reflect waves in that part of the spectrum for which the dispersion of a 60° rock-salt prism may be used to determine wave-lengths.

These oxides are tabulated below with their molecular weights and the wave-lengths which they are known to selectively reflect. In the column under W_m/W_0 are given the ratios of the weights of the metals to those of the oxygen with which they are combined. For convenience in making comparisons with the wave-lengths selectively reflected, the weights of the elements combined with 48 grams of oxygen, O_3, are added in the following table.

Substance.	Chemical Formula.	Molecular Weight.	W_m/W_0	$W_m/W_0\times 48$	Wave-lengths Selectively Reflected.
Ice.	H_2O	18.0	2.0/16	6	3.2μ
Quartz.	SiO_2	60.4	28.4/32	43	8.5μ– 9.0μ 12.5μ
Corundum.	Al_2O_3	102.2	54.2/48	54	11.0μ–11.8μ 13.5μ
Rutile.	TiO_2	80.1	48.1/32	72	13.6μ
Hematite.	Fe_2O_3	159.8	111.8/48	112	>13.6μ
Zincite.	ZnO	81.4	65.4/16	196	>13.6μ

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–23, 1910.

[2] Bode, Ann. der Physik, 30, p. 334, 1909. Coblentz, Jahr. der Rad. u. Elekt., 5, p. 1, 1908.

The data on the reflection of ice extend to 4μ and on the other oxides to about 14μ. The hematite and zincite show a nearly uniform reflection of 12 per cent. and 4 per cent. respectively between 1μ and 13μ. There were no data available which showed whether they selectively reflected longer waves.

Corundum and hematite are isomorphous. The other oxides are so different both chemically and crystallographically that they do not afford data suitable for a quantitative comparison,[1] but it was observed that as the ratio W_m/W_0 increased the shortest wave-length which the substance would selectively reflect increased. No relation was observed between the molecular weights and the wave-lengths selectively reflected.

[1]The data at hand are not sufficient to show whether isomorphous or only eutropic series of oxides, or of salts of oxygen acids, are suitable for such a comparison.